Module 1: Gas Power Cycles

Introduction

An important application of thermodynamics is the analysis of power cycles through which the energy absorbed as heat can be continuously converted into mechanical work. A thermodynamic analysis of the heat engine cycles provides valuable information regarding the design of new cycles or for improving the existing cycles.

The purpose of a thermodynamic cycle is either to produce power, or to produce refrigeration/pumping of heat. Therefore, the cycles are broadly classified as follows:

1. Heat engine or power cycles.
2. Refrigeration/heat pump cycles.

Any thermodynamic cycle is essentially a closed cycle in which, the working substance undergoes a series of processes and is always brought back to the initial state. However, some of the power cycles operate on open cycle. It means that the working substance is taken into the unit from the atmosphere at one end and is discharged into the atmosphere after undergoing a series of processes at the other end.

Analysis of Cycles

In air standard analysis, air is considered as the working medium. The analysis is carried out with the following assumptions.

Assumptions

1. The working substance consists of a fixed mass of air and behaves as a perfect gas. The closed system is considered which under goes a cycle process. Therefore, there are no intake or exhaust process.
2. The combustion process is replaced by an equivalent heat addition process form an external source. Thus there is no change in the chemical equilibrium of the working fluid and also composition.
3. There is no exhaust process; this is replaced by an equivalent heat rejection process.
4. Compression and expansion processes in the cycle are considered as reversible adiabatic process.
5. The specific heats C_p and C_v of air remains constant and does not vary with temperature.

Carnot Cycle

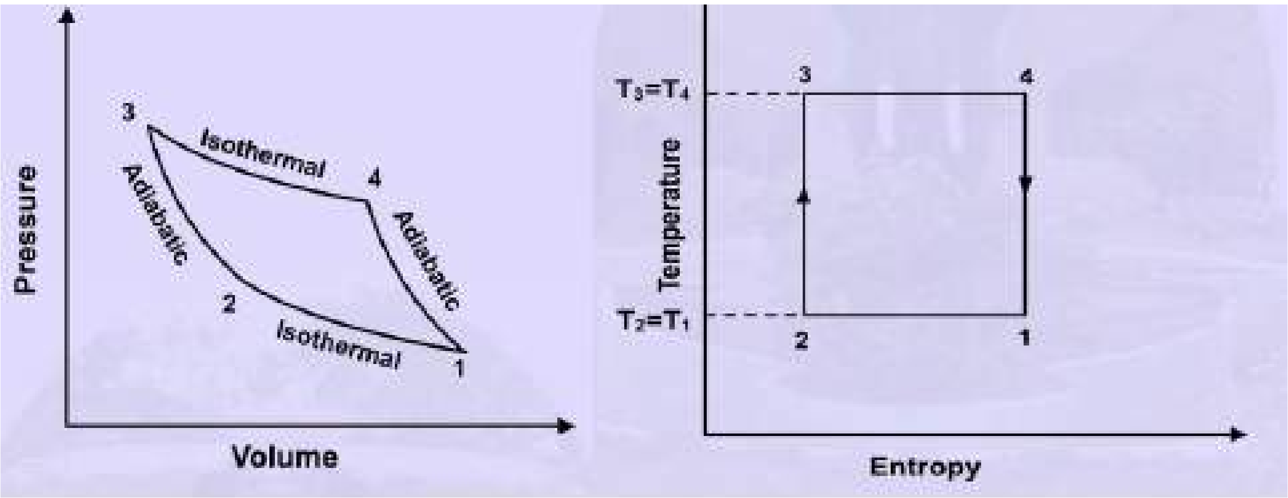


Figure 1: P-V and T-S diagram of Carnot Cycle

The T-s and p-v diagrams for a Carnot power cycle are shown in Fig.1. The cycle consists of two reversible adiabatic and two reversible isothermal processes, The working of the cycle is as follows:

Process 1-2: Reversible isothermal heat rejection of the working substance from state 1 to state 2.

Process 2-3: Isentropic compression of the working substance from state 2 to state 3. During this process work is done on the working substance by the surroundings.

Process 3-4: Reversible isothermal heat addition of the working substance from state 3 to state 4.

Process 4-1: Isentropic expansion of the working substance so that it comes back to its initial state. During this process work is done by the working substance on the surroundings.

Expression for Thermal Efficiency

From the thermodynamics, for unit mass of air
Heat supplied from point 3 to 4

$$p_3 v_3 \ln \frac{v_4}{v_3} \tag{1}$$

Heat rejected from point 1 to 2

$$p_1 v_1 \ln \frac{v_2}{v_1} \tag{2}$$

From characteristic gas equation we can write $p_3 v_3 = RT_{max}$ and $p_1 v_1 = RT_{min}$
Substituting in the above equations, we get

$$RT_{min} \ln \frac{v_2}{v_1}$$

$$RT_{max} \ln \frac{v_4}{v_3}$$

So the Work-done during the cycle is given by W = Heat supplied - Heat rejected

$$W = RT_{max} \ln \frac{v_4}{v_3} - RT_{min} \ln \frac{v_1}{v_2} \tag{3}$$

As the compression ratio

$$r_c = \frac{v_4}{v_3} = \frac{v_1}{v_2}$$

Substituting the above in the Work-done equation, we get

$$W = RT_{max} \ln r - RT_{min} \ln r$$

Carnot Thermal Efficiency is given by $= \dfrac{\text{Workdone}}{\text{Heat Supplied}}$
So, Mathematically it can written as,

$$\eta_{th} = \frac{RT_{max} \ln r - RT_{min} \ln r}{RT_{max} \ln r} \rightarrow \frac{T_{max} - T_{min}}{T_{max}} \tag{4}$$

Carnot cycle can be executed in a closed system (a piston and cylinder device or in a steady flow device. It can be seen that the thermal efficiency depends only on two temperatures T_{max} and T_{min} and is independent of working substance. The Carnot cycle is the most efficient cycle that can be executed between a heat source at temperature T_{max} and a heat sink at temperature T_{min}. But reversible isothermal heat transfer process is difficult to achieve in practice, because, it would require very large heat exchangers and it would take a very long time (a power cycle in a typical engine has to be completed in a fraction of a second). Therefore it is not practical to build an engine that would operate on a cycle that closely approximates a Carnot cycle.

The real value of the Carnot cycle comes from the fact that it is used as a standard against which the actual or other ideal power cycles are compared. It can be seen from the equation that the thermal efficiency the

Carnot power cycle increases with increase in T_{max} and with decrease in T_{min}. Hence in actual or other ideal cycles attempts are made in increasing the average temperature at which heat is supplied or by decreasing the average temperature at which heat is rejected. It should also be noted that the source and sink temperatures that can be used in practice have their limitations. The highest temperature in the cycle is limited by the maximum temperature the components of the engine can withstand and the lowest temperature is limited by the temperature of the cooling medium used in the cycle such as the atmospheric air, ocean, lake or a river.

Air Standard Otto Cycle

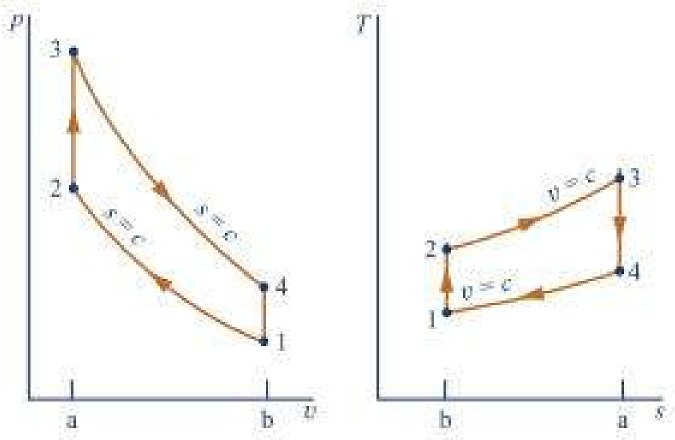


Figure 2: P-V and T-S diagram of Otto Cycle

Otto cycle is the ideal cycle for spark ignition engines. The cycle is named after Nikolaus A Otto, a German who built a four – stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862.The p–V and T–s diagrams for an Otto cycle are shown in Fig.2. The cycle consists of the following processes:

Process 1-2: Isentropic compression of air from state 1 to state 2. During this process. Work is done on air by the surroundings.

Process 2-3: Constant volume of heating of air from state 2 till the maximum permissible temperature is reached.

Process 3–4: Isentropic expansion of air from state 3 to state 4. During this process work is done by air on the surroundings.

Process 4–1: Constant volume cooling of air till the air comes back to its original state.

Expression for Thermal Efficiency and Work output

No heat is added during the isentropic compression processes 1-2 and 3-4.

Heat added during the constant volume heating process 2-3 is given by,

$$Q_H = c_v * (T_3 - T_2) \tag{5}$$

Heat rejected during the constant volume heating process 4-1 is given by,

$$Q_L = c_v * (T_1 - T_4) \tag{6}$$

Work done during the cycle is given by,

$$W = Q_H - Q_L = c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

Thermal Efficiency is given by, $\eta_{th} = \frac{W}{Q_H}$

$$\eta_{th} = \frac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)} \tag{7}$$

Further simplified as,

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$
$$\eta_{th} = 1 - \frac{1}{\frac{(T_3-T_2)}{(T_4-T_1)}} \tag{8}$$

The equation 8, gives the expression for thermal efficiency of the Otto cycle in terms of the temperatures at the salient points of the cycles. It is possible to express the net work output and thermal efficiency of the Otto cycle in terms of compression ratio. So, for the isentropic processes 1-2 and 3-4, we can write,

$$\frac{T_3}{T_4} = (\frac{v_4}{v_3})^{(\gamma-1)} \quad \text{and} \quad \frac{T_2}{T_1} = (\frac{v_1}{v_2})^{(\gamma-1)}$$

But $v_2 = v_3$ and $v_1 = v_4$ Therefore we can write,

$$\frac{(T_3 - T_2)}{(T_4 - T_1)} = \frac{(T_4(\frac{v_1}{v_2})^{(\gamma-1)} - T_1(\frac{v_1}{v_2})^{(\gamma-1)})}{(T_4 - T_1)} = \frac{(T_4 - T_1)}{(T_4 - T_1)}(\frac{v_1}{v_2})^{(\gamma-1)}$$
$$\frac{(T_3 - T_2)}{(T_4 - T_1)} = (\frac{v_1}{v_2})^{(\gamma-1)}$$

So the equation 8 for thermal efficiency can be written as

$$\eta_{th} = 1 - \frac{1}{(\frac{v_1}{v_2})^{(\gamma-1)}} = 1 - \frac{1}{r_c^{(\gamma-1)}} \tag{9}$$

From the above equation, it can be observed that the efficiency of the Otto cycle is mainly the function of compression ratio for the given ratio of C_p and C_v . If we plot the variations of the thermal efficiency with increase in compression ratio for different gases, the curves are obtained as shown in Fig.3, beyond certain values of compression ratios, the increase in the thermal efficiency is very small, because the curve tends to be asymptotic. However, practically the compression ratio of petrol engines is restricted to maximum of 9 or 10 due to the phenomenon of knocking at high compression ratios.

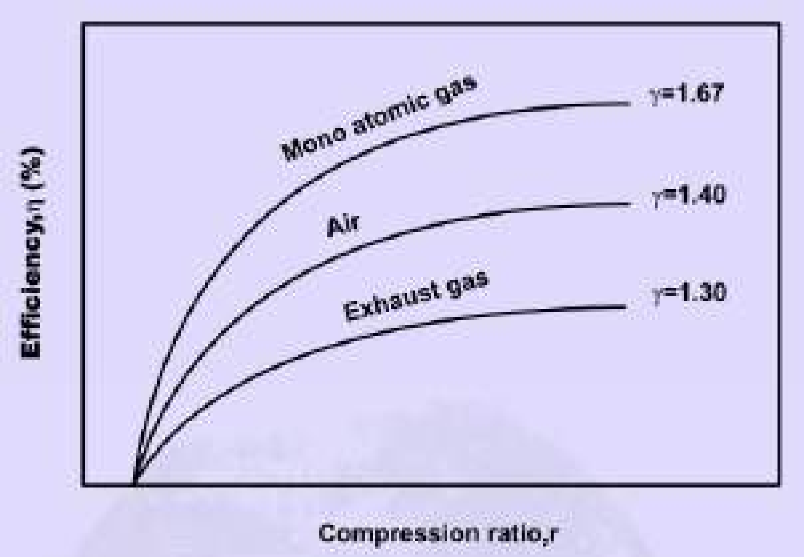


Figure 3: Variation of thermal efficiency with compression ratio

Expression for Mean effective pressure

Generally it is defined as the ratio of new workdone to the displacement volume of the pistion. Let us consider unit kg of working substance, Work done is given by,

$$W = c_v(T_3 - T_2) - c_v(T_4 - T_1) \tag{10}$$

Displacement volume is given by,

$$\begin{aligned}
v_s &= (v_1 - v_2) \\
v_s &= v_1(1 - \frac{1}{r}), \quad \text{But,} v_1 = \frac{RT_1}{p_1} \\
v_s &= \frac{RT_1}{p_1}(1 - \frac{1}{r}) \\
v_s &= \frac{c_v(\gamma - 1)T_1}{p_1}(\frac{r-1}{r}) \quad \text{Since,} \quad R = c_v(\gamma - 1) \qquad (11)
\end{aligned}$$

Mean effective pressure is given by;

$$\begin{aligned}
p_m &= \frac{(T_3 - T_2) - (T_4 - T_1)}{\frac{c_v(\gamma-1)T_1}{p_1}(\frac{r-1}{r})} \\
p_m &= \frac{1}{c_v(\gamma - 1)}\frac{p_1}{T_1}\frac{r}{r-1}[(T_3 - T_2) - (T_4 - T_1)] \qquad (12)
\end{aligned}$$

As $T_2 = T_1(r)^{\gamma-1)}$. Let Pressure ratio is $r_p = \frac{p_3}{p_2} = \frac{T_3}{T_2}$ and $T_3 = \frac{p_3}{p_2}T_2 = r_pT_2 = r_pT_1(r)^{(\gamma-1)}$

$$\begin{aligned}
\frac{T_4}{T_3} &= (\frac{v_3}{v_4})^{(\gamma-1)} = (\frac{1}{r})^{(\gamma-1)} \\
T_4 &= r_pT_1(r)^{(\gamma-1)} * (\frac{1}{r})^{(\gamma-1)} \\
T_4 &= r_pT_1 \qquad (13)
\end{aligned}$$

So the mean effective pressure is given by

$$\begin{aligned}
p_m &= \frac{p_1 r}{c_v(r-1)(\gamma-1)}[(r_p r^{\gamma-1} - r^{\gamma-1}) - (r_p - 1)] \\
p_m &= p_1 r[\frac{(r^{\gamma-1}(r_p - 1) - (r_p - 1)}{c_v(r-1)(\gamma-1)}] \\
p_m &= p_1 r[\frac{(r^{\gamma-1} - 1)(r_p - 1)}{c_v(r-1)(\gamma-1)}] \qquad (14)
\end{aligned}$$

The above expression is the equation for Mean effective pressure in terms of compression ration r, pressure ratio r_p and specific heat ratio γ

Air Standard Diesel Cycle

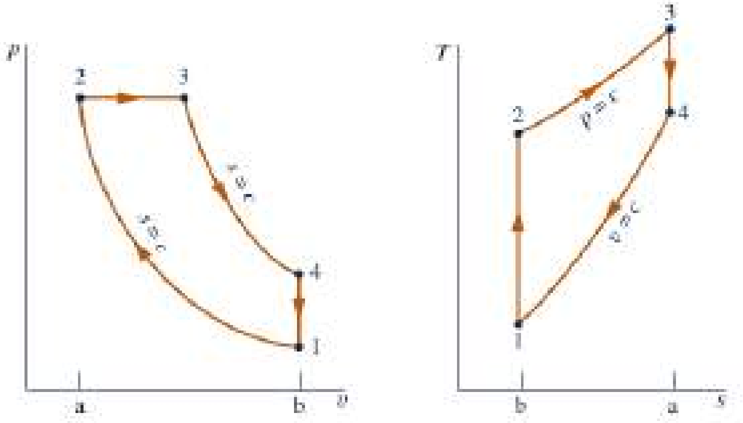


Figure 4: P-V and T-S diagram of Diesel Cycle

The diesel cycle is the ideal cycle for compression ignition engines (CI engines). CI engine was first proposed by Rudolph Diesel in 1890. The diesel engine works on the principle of compression ignition. In such an engine, only air is compressed and at the end of the compression process, the fuel is sprayed into the engine cylinder containing high pressure air, so that the fuel ignites spontaneously and combustion occurs. Since only air is compressed during the compression stroke, the possibility of auto ignition is completely eliminated in diesel engines. Hence diesel engines can be designed to operate at much higher compression ratios (between 12 and 24). Also another benefit of not having to deal with auto ignition is that fuels used in this engine can be less refined (thus less expensive).

Air standard diesel cycle is a idealized cycle for diesel engines. It is as shown on P-v and T-s diagrams. The processes in the cycle are as follows:

Process 1-2: Reversible adiabatic Compression.During this process the work is done on the air by the surroundings.

Process 2-3: Constant pressure heat addition till the maximum permissible temperature is reached.

Process 3-5: Reversible adiabatic Compression. During this process work is done by the air on the surroundings.

Process 4-1: Constant volume heat rejection so that air comes back to its original state to complete the cycle.

Expression for Work output and Thermal Efficiency

Consider unit kg of working fluid. Since the compression and expansion processes are reversible adiabatic processes, we can write,

$$\begin{aligned} \text{Heat supplied} \quad Q_H &= c_p(T_3 - T_2) \\ \text{Heat Rejected} \quad Q_L &= c_v(T_1 - T_4) \end{aligned}$$

Work done during the cycle is given by,

$$\begin{aligned} W &= Q_H - Q_L \\ W &= c_p(T_3 - T_2) - c_v(T_4 - T_1) \end{aligned}$$

Now, we can write Thermal efficiency as,

$$\begin{aligned} \eta_{th} &= \frac{W}{Q_H} \\ \eta_{th} &= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} \\ \eta_{th} &= 1 - \frac{1}{\gamma}(\frac{T_4 - T_1}{T_3 - T_2}) \end{aligned} \tag{15}$$

The above expression gives the equation for thermal efficiency in terms of temperatures and ratio of specific heat γ. We can get the expression for thermal efficiency in terms of compression ratio r_c, cut-off ratio ρ. Now expressing all the Temperatures in terms of T_1, γ and r_c, we get.,

$$\begin{aligned} & T_2 = T_1 r_c^{\gamma-1} \\ & \text{such that,} r = \frac{v_1}{v_2} \text{also cut-off ratio } \ \rho = \frac{v_3}{v_2} = \frac{T_3}{T_2} \\ & T_3 = \rho T_2 = \rho T_1 r_c^{\gamma-1} \\ & \frac{T_4}{T_3} = (\frac{v_3}{v_4})^{\gamma-1} \\ & T_4 = \rho T_1 r_c^{\gamma-1}(\frac{v_3 v_2}{v_2 v_4})^{\gamma-1} \\ & T_4 = \rho T_1 r_c^{\gamma-1}(\frac{\rho}{r_c})^{\gamma-1} = \rho^{\gamma} T_1 \end{aligned}$$

Substituting the above temperature expressions in equation 23, we get

$$\eta_{th} = 1 - \frac{1}{\gamma}\frac{\rho^{\gamma}T_1 - T_1}{\rho r_c^{\gamma-1}T_1 - r_c^{\gamma-1}T_1}$$
$$\eta_{th} = 1 - \frac{\rho^{\gamma} - 1}{\gamma r_c^{\gamma-1}(\rho - 1)} \tag{16}$$

From the above equation, it is observed that the thermal efficiency of the diesel engine can be increased by increasing the compression ratio r_c, by decreasing the cut-off ratio ρ and by using a large value of γ. Since the quantity, $\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}$ is always greater than unity, the efficiency of the diesel cycle is always lower than that of an Otto cycle having the same compression ratio. However, practical Diesel engines uses higher compression ratios compared to petrol engines.

Expression for Mean effective pressure

As already discussed during Otto cycle, it is defined as the ratio of new workdone to the displacement volume of the pistion. Let us consider unit kg of working substance, Heat supplied during the diesel cycle is given,

$$Q_H = c_p(T_3 - T_2)$$

Heat rejected during the diesel cycle is given,

$$Q_L = c_v(T_1 - T_4)$$

Work done is given by,

$$W = Q_H - Q_L = c_p(T_3 - T_2) - c_v(T_4 - T_1) \tag{17}$$

Displacement volume is given by,

$$v_s = (v_1 - v_2)$$
$$v_s = v_1(1 - \frac{1}{r_c}), \quad \text{But, } v_1 = \frac{RT_1}{p_1}$$
$$v_s = \frac{RT_1}{p_1}(1 - \frac{1}{r_c})$$
$$v_s = \frac{c_v(\gamma-1)T_1}{p_1}(\frac{r_c - 1}{r_c}) \quad \text{Since,} \quad R = c_v(\gamma - 1) \tag{18}$$

Mean effective pressure is given by;

$$p_m = \frac{\gamma(T_3 - T_2) - (T_4 - T_1)}{\frac{c_v(\gamma-1)T_1}{p_1}(\frac{r_c-1}{r_c})}$$
$$p_m = \frac{1}{c_v(\gamma-1)}\frac{p_1}{T_1}\frac{r_c}{r_c - 1}[\gamma(T_3 - T_2) - (T_4 - T_1)] \tag{19}$$

As $T_2 = T_1(r_c)^{\gamma-1)}$, if Cut-off ratio $\rho = \frac{v_3}{v_2}$,r_e is the expansion ratio $= \frac{v_4}{v_3} = \frac{v_4 v_2}{v_2 v_3}$ so expansion ratio $r_e = \frac{r_c}{\rho}$
Then, $T_3 = \frac{v_3}{v_2}T_2 = \rho T_2 = \rho T_1(r_c)^{(\gamma-1)}$

$$\frac{T_4}{T_3} = (\frac{v_3}{v_4})^{(\gamma-1)} = \frac{1}{r_e^{\gamma-1}} = (\frac{\rho}{r_c})^{(\gamma-1)}$$
$$T_4 = \rho T_1(r_c)^{(\gamma-1)} * (\frac{\rho}{r_c})^{(\gamma-1)}$$
$$T_4 = \rho^{\gamma}T_1 \tag{20}$$

So the mean effective pressure is given by

$$p_m = \frac{p_1 r_c}{c_v(r_c - 1)(\gamma - 1)}[\gamma(\rho r_c^{\gamma-1} - r_c^{\gamma-1}) - (\rho^{\gamma} - 1)]$$
$$p_m = p_1 r_c[\frac{\gamma r_c^{\gamma-1}(\rho - 1) - (\rho^{\gamma} - 1)}{c_v(r_c - 1)(\gamma - 1)}] \tag{21}$$

The above expression is the equation for Mean effective pressure in terms of compression ration r, cut-off ratio ρ and specific heat ratio γ

Difference between Actual Diesel and Otto cycles

Otto Cycle

1. Homogeneous mixture of fuel and air formed in the Carburettor is supplied to the engine cylinder.
2. Ignition is initiated by means of electric spark plug.
3. Power output is varied by means of throttle-valve near Carburettor, which controls the fuel-air mixutre supply to the engine

Diesel Cycle

1. No carburetor is used. Air alone is supplied to the engine cylinder. Fuel is injected directly into the engine cylinder at the end of compression stroke by means of a fuel injector. Fuel-air mixture is heterogeneous.
2. No spark plug is used. Compression ratio is high and the high temperature of air ignites fuel.
3. No throttle value is used. Power output is controlled only by means of the mass of fuel injected by the fuel injector.

Air Standard Dual-Combustion or Limited Pressure or Semi-Diesel cycle

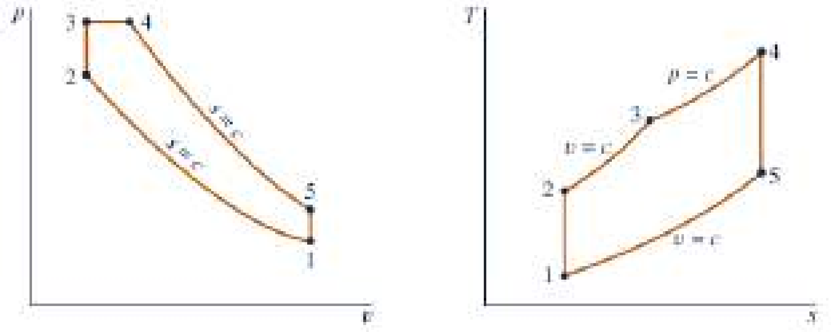


Figure 5: P-V and T-S diagram of Dual Cycle

The cycle is also called as the dual cycle, which is show in Figure. 5. Here the heat addition occurs partly at constant volume and partly at constant pressure. This cycle is a closer approximation to the behaviour of the actual Otto and Diesel engines because in the actual engines, the combustion process doesn't exactly occur at constant ovolume or at constant pressure but rather as in the dual cycle. Hence for most oil engines the ideal cycle is taken as the dual cycle. The P-V and T-S diagram for the dual cycle is shown above and it consists of follow processes,
Process 1-2: Reversible adiabatic compression.
Process 2-3: Constant volume heat addition.
Process 3-4: Constant pressure heat addition.
Process 4-5: Reversible adiabatic expansion.
Process 5-1: Constant volume heat rejection.
Consider unit kg of mass during the cyclic process,
Heat supplied during the process 2-3 and 3-4 is given by,

$$Q_H = c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

Heat rejected during the process 5-1 is given by,

$$Q_L = c_v(T_1 - T_5)$$

Thermal efficiency of the cycle can be obtained by the equation
$\eta_{th} = \dfrac{\text{Work done}}{\text{Heat Supplied}}$

$$\begin{aligned}
\eta_{th} &= \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \\
\eta_{th} &= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)} \\
\eta_{th} &= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \qquad (22)
\end{aligned}$$

The efficiency of the cycle can be expressed in terms of the following ratios, compression ratio $r_c = \frac{v_1}{v_2}$, Expansion ratio $r_c = \frac{v_5}{v_4}$, Cut-off ratio $\rho = \frac{v_4}{v_3}$ and pressure ratio $r_p = \frac{p_3}{p_2}$
We can express, $r_c = \rho * r_e$ such that $r_e = \frac{r_c}{\rho}$
For the process 3-4

$$\begin{aligned}
\rho &= \frac{v_4}{v_3} = \frac{T_4 p_3}{p_4 T_3} = \frac{T_4}{T_3} \\
T_3 &= \frac{T_4}{\rho} \qquad (23)
\end{aligned}$$

For the process 2-3

$$\begin{aligned}
\frac{p_2 v_2}{T_2} &= \frac{p_3 v_3}{T_3} \\
T_2 &= T_3 \frac{p_2}{p_3} = \frac{T_4}{r_p \rho} \qquad (24)
\end{aligned}$$

For the process 1-2

$$\begin{aligned}
\frac{T_2}{T_1} &= (\frac{v_1}{v_2})^{\gamma-1} \\
T_2 &= T_1(r_c)^{\gamma-1} \\
T_1 &= \frac{T_4}{r_p.\rho.r_c^{\gamma-1}} \qquad (25)
\end{aligned}$$

For the process 4-5

$$\begin{aligned}
\frac{T_5}{T_4} &= (\frac{v_4}{v_5})^{\gamma-1} = \frac{1}{r_e^{\gamma-1}} \\
T_5 &= T_4 \frac{\rho^{\gamma-1}}{r_c^{\gamma-1}} \qquad (26)
\end{aligned}$$

Substituting the values of $T_1, T_2, T_3 and T_5$ in the expression for the efficiency equation, we get

$$\begin{aligned}
\eta_{th} &= 1 - \frac{T_4 \dfrac{\rho^{\gamma-1}}{r_c^{\gamma-1}} - \dfrac{T_4}{r_p.\rho.r_c^{\gamma-1}}}{(\frac{T_4}{\rho} - \frac{T_4}{r_p.\rho} + \gamma(T_4 - \dfrac{T_4}{\rho}}) \\
\eta_{th} &= 1 - \frac{1}{r_c^{\gamma-1}} . \frac{r_p.\rho - 1}{(r_p - 1) + \gamma.r_p(\rho - 1)} \qquad (27)
\end{aligned}$$

Comparison on Otto, Diesel and Dual cycles

The important variable factors which are used as the basis for comparison of the cycles are compression ratio, peak pressure, heat rejection and the net work. In order to compare the performance of the Otto, Diesel and Dual combustion cycles, some of the variable factors must be fixed. In this section, a comparison of these three cycles is made for the same compression ratio, constant maximum pressure and temperature, same heat rejection and net work output. This analysis will show which cycle is more efficient for a given set of operating conditions.

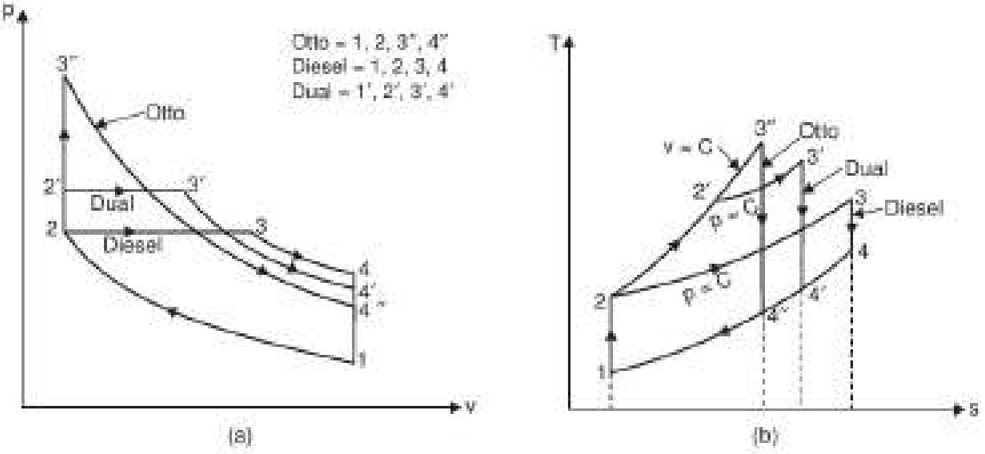


Figure 6: P-V and T-S diagram on same compression ratio and heat input

Case:1 Same compression ratio and heat input

A comparison of the cycles (Otto, Diesel and Dual) on the p-v and T-s diagrams for the same compression ratio and heat supplied is shown in the Fig.6.
We know that, $\eta = 1 - \frac{HeatSupplied}{HeatRejected}$
Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram. As is evident from the efficiency equation, the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.
i.e., $\eta_{otto} > \eta_{dual} > \eta_{diesel}$.

Case-2: Same Maximum Temperature, pressure and Heat rejection

The air-standard Otto, Dual and Diesel cycles are drawn on common p-v and T-s diagrams for the same maximum pressure and maximum temperature, for the purpose of comparison.

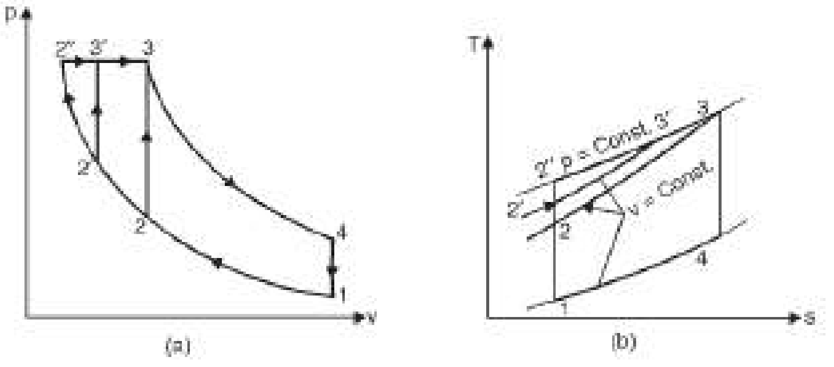


Figure 7: P-V and T-S diagram for comparison on same Maximum Temp, pressure and Heat rejection

$Otto\ 1-2-3-4-1, Dual\ 1-2^{'}-3^{'}-3-4-1, Diesel\ 1-2^{"}-3-4-1$ as shown in Fig.7. Slope of constant volume lines on T-s diagram is higher than that of constant pressure lines.
Here the otto cycle must be limited to a low compression ratio(r_c) to fulfill the condition that point 3 (same maximum pressure and temperature) is to be a common state for all the three cycles. The construction of cycles on T-s diagram proves that for the given conditions the heat rejected is same for all the three cycles (area under process line 4-1).
Since, by definition, $\eta = 1 - \frac{HeatSupplied}{HeatRejected} = 1 - \frac{Constant}{Q_H}$
the cycle with greater heat addition will be more efficient. From the T-S diagram
$Q_{H-Diesel}$= Area under $2^{"}-3$, Q_{H-Dual}= Area under $2^{'}-3'-3$, Q_{H-Otto}= Area under $2-3$
It can be seen that $Q_{H-Diesel} > Q_{H-Dual} > Q_{H-Otto}$
and thus efficiencies are given by, $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$.

Brayton Cycle

Gas turbine engines operate on either open or closed basis. The Fig. 8a is the open mode of operation which is commonly used. In this type the atmospheric air is continuously drawn into the compressor and compressed to a high

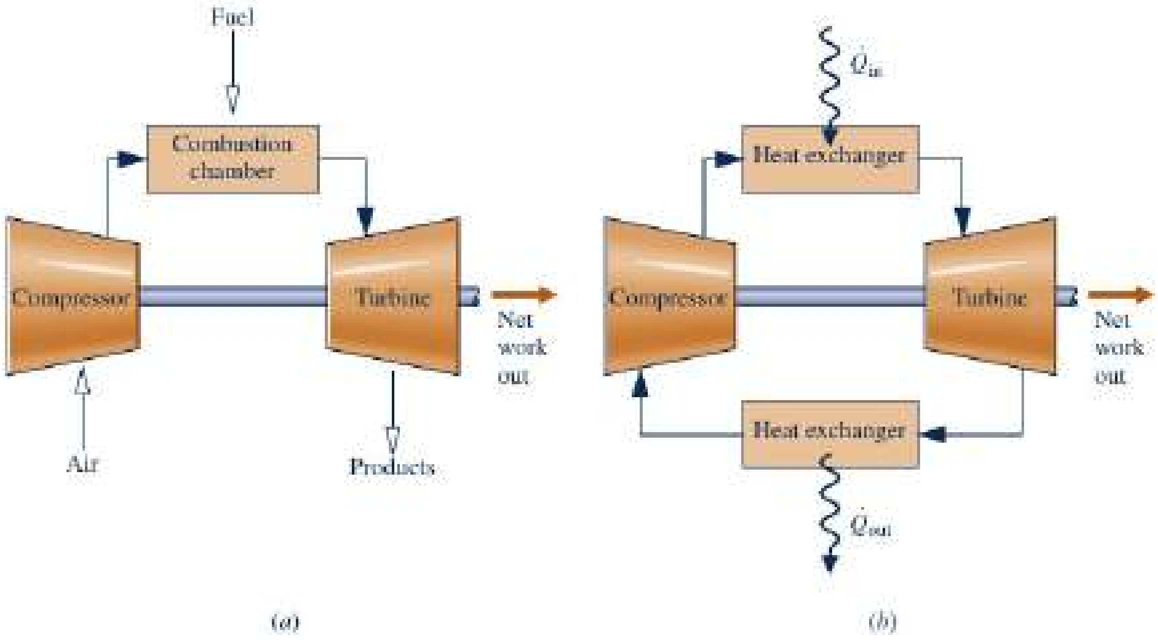


Figure 8: Simple Gasturbine (a) Open cycle (b) Close cycle

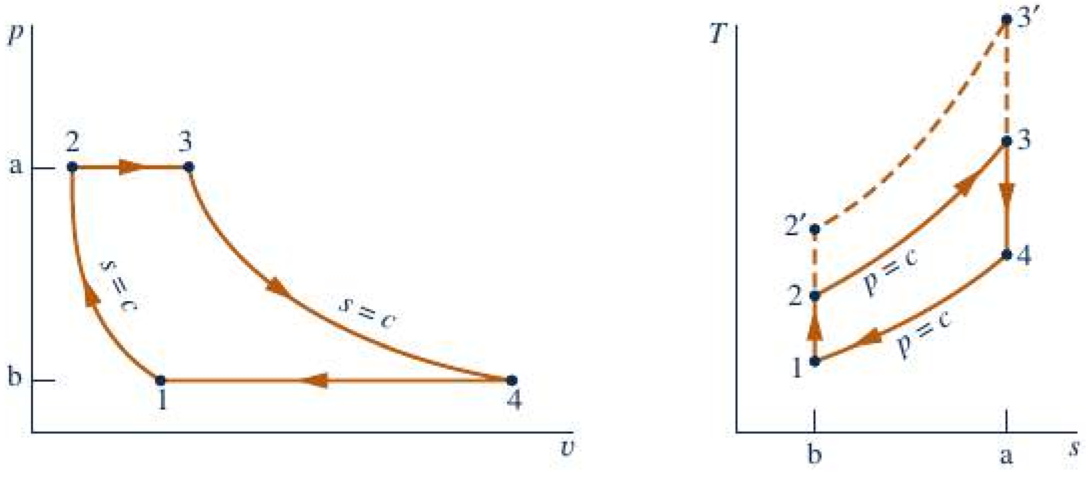
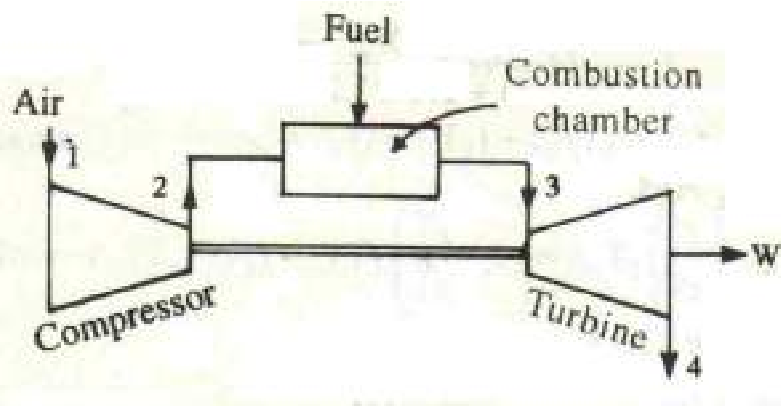


Figure 9: P-V and T-S diagram for Brayton cycle

GAS TURBINES

Gas Turbines and Jet Propulsion: Classification of Gas Turbines, Analysis of open cycle gas turbine cycle. Advantages and Disadvantages of closed cycle. Methods to improve thermal efficiency. Jet propulsion and Rocket propulsion.

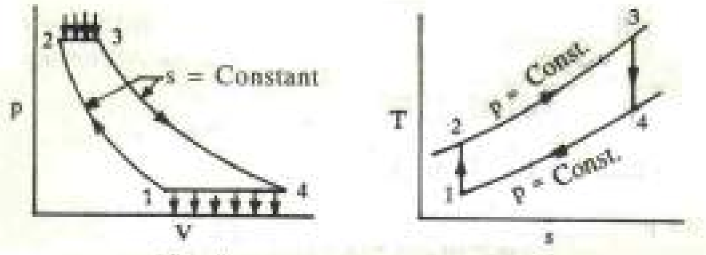
Simple Gas Turbine Cycle



Open type gas-turbine cycle

A schematic diagram of a simple gas turbine power plant is shown in figure. Air is drawn from the atmosphere into the compressor, where it is compressed reversibly and adiabatically. The relatively high pressure is then used in burning the fuel in the combustion chamber. The air fuel ratio is quite high (about 60:1) to limit the temperature of the burnt gases entering the turbine. The gases then expand isentropically in the turbine. A portion of the work obtained from the turbine is utilized to drive the compressor and the auxiliary drive, and rest of the power output is the net power of the gas turbine plant.

A gas turbine plant works using a Brayton or joule cycle. This cycle was originated by joule, a British engineer for use in a hot air reciprocating engine and later in about 1870 an American engineer George Brayton tried this cycle in a gas turbine. This cycle consists of two constant pressures and two adiabatic processes. The P-V and T-S diagrams of the cycle are as shown in figure.



p-v and T-s diagrams of air stander gas-turbine cycle

Process 1 – 2: isentropic compression in the compressor

Process 2 – 3: constant pressure heat addition in the combustion chamber

Process 3 – 4: isentropic expansion in the turbine

Process 4 -1: constant pressure heat rejection in the atmosphere or cooling of air in the intercooler (closed cycle).

Expression of net work output:

We have net work output, $W_N = W_T - W_C$
Turbine work, $W_T = h_3 - h_4$
$= C_P (T_3 - T_4)$ since the working fluid is a perfect gas

Compressor work, $W_C = h_2 - h_1$
$= C_P (T_2 - T_1)$

$\therefore W_N = C_P (T_3 - T_4) - C_P (T_2 - T_1)$

Let $R = \frac{P_2}{P_1} =$ pressure ratio for compression

$t = T_3/T_1$ = Temperature ratio

$$W_N = C_P T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1 \right]$$

We have $\frac{T_1}{P_1^{\frac{r-1}{r}}} = \frac{T_2}{P_2^{\frac{r-1}{r}}}$ $\quad \therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \frac{T_3}{T_1}$$

$$= \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} t = \left(\frac{1}{R} \right)^{\frac{r-1}{r}} t \qquad \because P_1 = P_4 \quad \& \quad P_2 = P_3$$

$$\therefore W_N = C_P T_1 \left[t - \frac{t}{R^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} + 1 \right]$$

Expression for Thermal Efficiency:

We have thermal efficiency, $\eta_{th} = \frac{W_N}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$

Heat added, $Q_H = h_3 - h_2 = C_P\,(T_3 - T_2)$
Heat rejected, $Q_L = h_4 - h_1 = C_P\,(T_4 - T_1)$

$$\therefore \eta_{th} = 1 - \frac{C_P(T_4 - T_1)}{C_P(T_3 - T_2)} = 1 - \frac{T_1\left[\dfrac{T_4}{T_1} - 1\right]}{T_2\left[\dfrac{T_3}{T_2} - 1\right]}$$

Now, $\dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$ & $T_3/T_4 = \left(P_3/P_4\right)^{\frac{r-1}{r}} = \left(\dfrac{1}{R}\right)^{\frac{r-1}{r}}$

But as $P_2 = P_3$ & $P_1 = P_4$, it follows that $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_4}$ *or* $\dfrac{T_4}{T_1} = \dfrac{T_3}{T_1}$

$$\therefore \eta_{th} = 1 - \frac{T_1}{T_2} \quad \textit{i.e.,} \quad \eta_{th} = 1 - \frac{1}{\left(\dfrac{T_2}{T_1}\right)} \quad \text{or} \quad \eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}}$$

From the above equation, it is seen that the efficiency of the air standard gas turbine cycle increases with increase in pressure ratio (R) and the type of working fluid.

Optimum Pressure Ratio for Specific Power Output

In a gas turbine cycle, T_1 is the temperature of the atmosphere and T_3 is the temperature of the burnt gases entering the turbine. Temperature T_3 is fixed by the metallurgical consideration of the turbine and temperature T_1 is fixed by the atmospheric condition. Between these two extreme values of temperature, there exists an optimum pressure ratio for which the work output of the turbine is maximum.

We have, the net work output of the turbine is,

$$W_N = C_P T_1\left[t - \frac{t}{R^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} - 1\right] \qquad \text{--- (1)}$$

The optimum pressure ratio is obtained by differentiating the net work output w.r.t. the pressure ratio and putting the derivative equal to zero i.e., $\dfrac{dW_N}{dR} = 0$

Or $\dfrac{d}{dR}\left[C_P T_1\left\{t - \dfrac{t}{R^{\frac{\gamma-1}{\gamma}}} - R^{\frac{\gamma-1}{\gamma}} - 1\right\}\right] = 0$

Differentiating with respect to R we get,

$$-t\frac{1-\gamma}{\gamma}R^{\frac{1-\gamma-\gamma}{\gamma}}-\frac{\gamma-1}{\gamma}R^{\frac{\gamma-1-\gamma}{\gamma}}=0$$

i.e., $$-t\left(\frac{1-\gamma}{\gamma}\right)R^{\frac{1-2\gamma}{\gamma}}=\frac{\gamma-1}{\gamma}R^{-1/\gamma}$$

$$t\left(\frac{\gamma-1}{\gamma}\right)R^{\frac{1-2\gamma}{\gamma}}=\frac{\gamma-1}{\gamma}R^{-1/\gamma}$$

or $$\frac{R^{-1/\gamma}}{R^{\frac{1-2\gamma}{\gamma}}}=t \quad or \quad R^{-\frac{1}{\gamma}-\frac{1-2\gamma}{\gamma}}=t$$

or $$R^{\frac{-1-1+2\gamma}{\gamma}}=t \quad or \quad R^{\frac{2(\gamma-1)}{\gamma}}=t$$

or $$(R)_{opt}=t^{\frac{\gamma}{2(\gamma-1)}} \qquad \text{i.e., } R_{opt}=\left[\frac{T_3}{T_1}\right]^{\frac{\gamma}{2(\gamma-1)}}$$

Substituting this value of R in the expression for W_N, we get

$$(W_N)_{opt}=C_PT_1\left[t-\frac{t}{\left[t^{\frac{r}{2(r-1)}}\right]^{\frac{r-1}{r}}}-\left[t^{\frac{r}{2(r-1)}}\right]^{\frac{r-1}{r}}+1\right]$$

$$=C_PT_1\left[t-\frac{t}{t^{\frac{1}{2}}}-t^{\frac{1}{2}}+1\right]$$

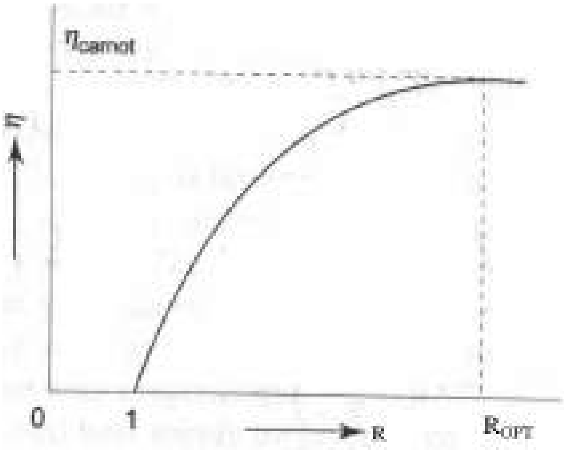
$$=C_PT_1\left[t-t^{\frac{1}{2}}-t^{\frac{1}{2}}+1\right]$$

$$=C_PT_1\left[t-2t^{\frac{1}{2}}+1\right]$$

$$(W_N)_{opt}=C_PT_1\left[t^{\frac{1}{2}}-1\right]$$

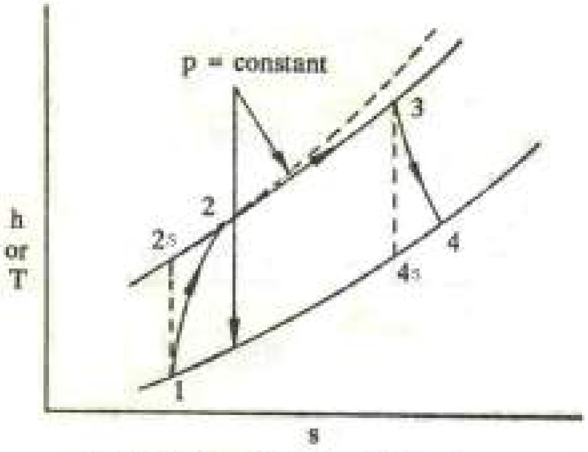
$$\eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}} = 1 - \frac{1}{\left[t^{\frac{r}{2(r-1)}}\right]^{\frac{r-1}{r}}}$$

$$\therefore (\eta_{th})_{opt} = 1 - \frac{1}{t^{\frac{1}{2}}}$$



Effect of pressure ratio on Brayton cycle efficiency

In an ideal gas turbine plant, the compression and expansion processes are isentropic and there is no pressure-drop in the combustion chamber. But because of irreversibilities associated in the compressor and the turbine, and the pressure-drop in the actual flow passages and combustion chamber, an actual gas turbine plant differs from ideal one. The T-S diagram of actual plant is shown in figure.



T-s diagram of the actual cycle

$$\therefore \textit{Compressor efficeincy}, \quad \eta_C = \frac{h_{2S} - h_1}{h_2 - h_1}$$

$$\text{and the } \textit{turbine efficeincy}, \quad \eta_t = \frac{h_3 - h_4}{h_3 - h_{4S}}$$

Classification: Gas turbine are mainly divided into two group

I Constant pressure combustion gas turbine
i) Open cycle, ii) Closed cycle

II Constant volume combustion gas turbine
In almost all the field open cycle gas turbine plants are used. Closed cycle plants were introduced at one stage because of their ability to burn cheap fuel.

Advantages and disadvantages of closed cycle over open cycle

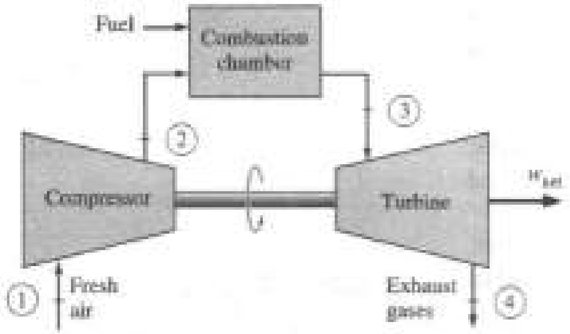
Advantages of closed cycle:

i) Higher thermal efficiency
ii) Reduced size
iii) No contamination
iv) Improved heat transmission
v) Improved part load η
vi) Lesser fluid friction
vii) No loss of working medium
viii) Greater output and
ix) Inexpensive fuel.

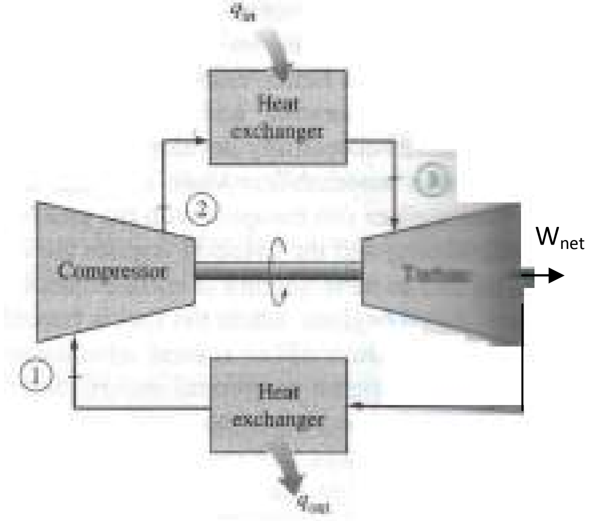
Disadvantages of closed cycle:

i) Complexity
ii) Large amount of cooling water is required. This limits its use of stationary installation or marine use
iii) Dependent system
iv) The wt of the system pre kW developed is high comparatively, ∴ not economical for moving vehicles
v) Requires the use of a very large air heater.

An Open cycle and Closed cycle Gas Turbine Engines



An open-cycle gas-turbine engine.

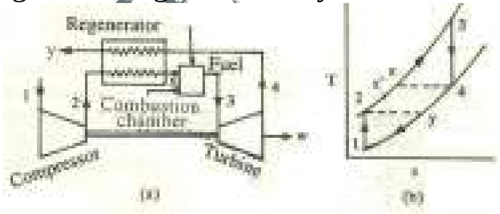


A closed-cycle gas-turbine engine

Methods to improve the performance of simple gas turbine plants

I Regenerative Gas Turbine Cycle: The temperature of the exhaust gases in a simple gas turbine is higher than the temperature of the air after compression process. The η of the Brayton cycle can be increased by utilizing part of the energy of the exhaust gas from the turbine in heating up the air leaving the compression in a heat exchanger called a regenerator, thereby reducing the amount of heat supplied from an external source and also the amount of heat rejected.

Figure shows a single stage regenerative gas turbine cycle



An ideal air standard regenerative gas turbine cycle

Air is drawn from the atmosphere into the compressor and is compressed isentropically to state 2. It is then heated at constant pressure in the regenerator to state x by the hot burnt gases from the gas turbine. Since the temperature of the air increases before it reaches the combustion chamber, less amount of fuel will be required to attain the designed turbine inlet temperature of the products of combustion. After combustion at constant pressure in the combustion chamber, the gas enters the turbine at state 3 and expands isentropically to state 4 in the turbine. It then enters the counter-flow regenerator as stated earlier, where it gives up a portion of its heat energy to the compressed air from the compressor and leaves the regenerator at state y.

In an ideal cycle, the temperature of the air leaving the regenerator is equal to the temperature of the burnt gases leaving the turbine, i.e., $T_x = T_4$. But in practice, the temperature of the air leaving the regenerator is less than T_x. In T-S diagram, T_x^1 is the temperature of the air leaving the regenerator in an actual plant.

$\therefore$ Effectiveness of a regenerator is $\varepsilon = \eta_r = \dfrac{T_{x^1} - T_2}{T_x - T_2}$ when C_P is constant.

In an ideal regenerator, heat loss by the burnt gases is equal to the heat gained by the air in the regenerator, i.e., $T_4 - T_y = T_x - T_2$,
Where $T_x = T_4$ and $T_y = T_2$ and hence $\eta_r = 1$

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

For an ideal regenerative gas turbine cycle,

$Q_L = C_P\,(T_y - T_1) = C_P\,(T_2 - T_1)$ and
$Q_H = C_P\,(T_3 - T_x) = C_P\,(T_3 - T_4)$

$$\therefore \eta_{th} = 1 - \frac{(T_2 - T_1)}{(T_3 - T_4)} = 1 - \frac{T_1\left[\frac{T_2}{T_1} - 1\right]}{T_3\left[1 - \frac{T_4}{T_3}\right]}$$

Since $\dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$ *and* $\dfrac{T_4}{T_3} = \left(\dfrac{P_4}{P_3}\right)^{\frac{r-1}{r}} = \left(\dfrac{P_1}{P_2}\right)^{\frac{r-1}{r}} = \left(\dfrac{1}{R}\right)^{\frac{r-1}{r}}$

$$\therefore \eta_{th} = 1 - \frac{1}{t}\frac{\left(R^{\frac{r-1}{r}} - 1\right)}{\left(1 - \frac{1}{R^{\frac{r-1}{r}}}\right)} = 1 - \frac{1}{t}.\quad R^{\frac{r-1}{r}}$$

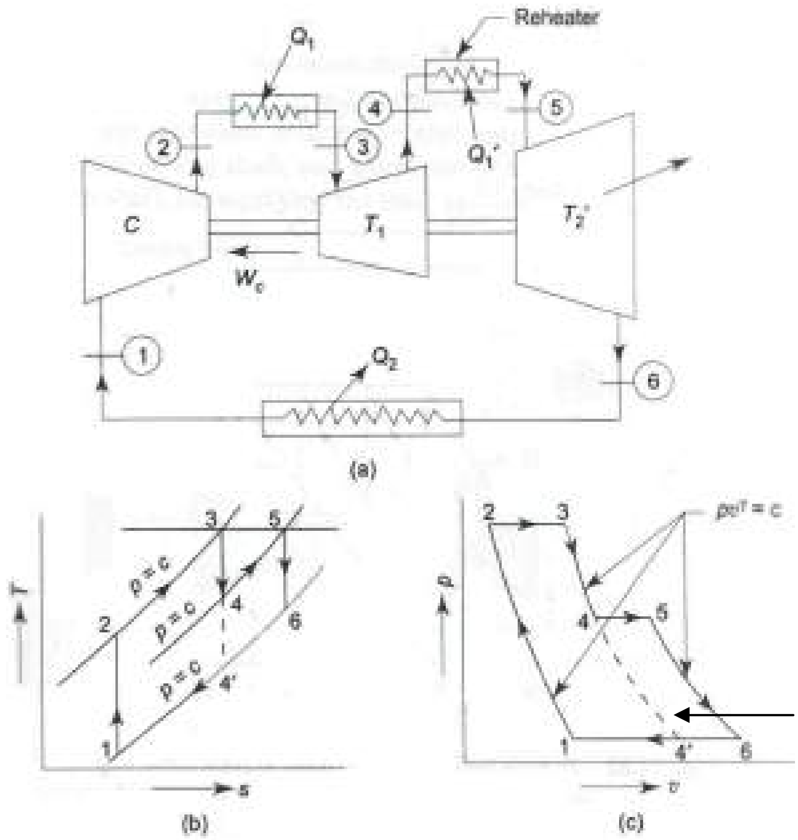
i.e., $\eta_{th} = 1 - \dfrac{1}{t} R^{\frac{r-1}{r}}$

It is evident that the η_{th} of an ideal regenerative gas turbine cycle depends not only on the pressure ratio but also on the ratio of the two extreme temperatures. For a fixed ratio of T_3/T_1, the cycle η drops with increasing pressure ratio.

In practice the regenerator is costly, heavy and bulky, and causes pressure losses which brings about a decrease in cycle η.

II Ideal regenerative cycle with inter cooling and reheat:

a) Gas turbine cycle with reheat and multistage expansion:

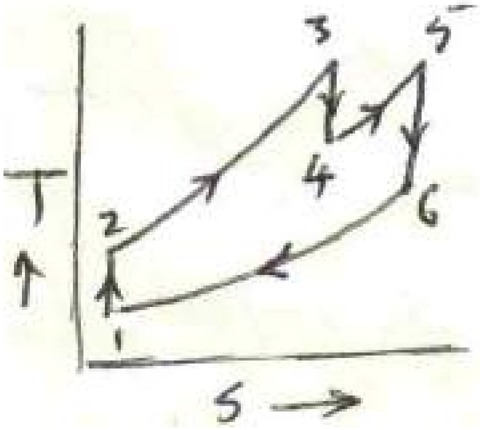


Increased work output due to reheating

Effect of reheat on Brayton cycle

Work output of the turbine is increased by multistage expansion with reheating. In the above illustration, two-stage expansion is done in low pressure (T_1) and high pressure (T_2^1) turbines with reheating the air in between stages.

Optimum work output for a two-stage reheat cycle:



Assumptions: The air after one stage of expansion is reheated back to its original temperature i.e., $T_3 = T_5$

Let $R = \frac{P_2}{P_1}, \quad R_1 = \frac{P_3}{P_4} \quad and \quad R_2 = \frac{P_5}{P_6}, t = \quad \frac{T_3}{T_1} = \max imum\ cycle\ temperature\ ratio$

$$\therefore R_1R_2 = \frac{P_3}{P_4}\frac{P_5}{P_6} \quad \because P_5 = P_4$$

$$= \frac{P_2}{P_1} = R$$

Net work output is $W_N = C_P\,(T_3 - T_4) + C_P\,(T_5 - T_6) - C_P\,(T_2 - T_1)$

$$= C_PT_1\left[\frac{T_3}{T_1} - \frac{T_4}{T_1} + \frac{T_3}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1\right]$$

we have $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3}\frac{T_3}{T_1} = \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}}\frac{T_3}{T_1} = \quad t\Big/R_1^{\frac{r-1}{r}}$$

$$\frac{T_6}{T_1} = \frac{T_6}{T_5}\frac{T_5}{T_1} = \left(\frac{P_6}{P_5}\right)^{\frac{r-1}{r}}\frac{T_3}{T_1} = \quad t\Big/R_2^{\frac{r-1}{r}}$$

$$\therefore W_N = C_PT_1\left[t - \frac{t}{R_1^{\frac{r-1}{r}}} + t - \frac{t}{R_2^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} + 1\right]$$

but $R_2 = R/R_1$

$$\therefore W_N = C_PT_1\left[2t - \frac{t}{R_1^{\frac{r-1}{r}}} - t\left(\frac{R_1}{R}\right)^{\frac{r-1}{r}} - R^{\frac{r-1}{r}} + 1\right]$$

For given values of R, t and T_1, W_N is maximum if $\frac{dW_N}{dR_1} = 0$

$$\therefore \frac{dW_R}{dR_1} = C_PT_1\left[0 - t.\frac{1-\gamma}{\gamma}.R_1^{\frac{\gamma-1}{\gamma}-1} - \frac{t}{R^{\frac{\gamma-1}{\gamma}}}.\frac{\gamma-1}{\gamma}.R_1^{\frac{\gamma-1}{\gamma}-1} - 0 + 0\right] = 0$$

i.e., $\frac{\gamma-1}{\gamma}.t\ R_1^{\frac{1-2\gamma}{\gamma}} - \frac{\gamma-1}{\gamma}\frac{t}{R^{\frac{\gamma-1}{\gamma}}} \quad R_1^{-1/\gamma} = 0$

$$R_1^{\frac{1-2r}{r}} = \frac{R_1^{-\frac{1}{\gamma}}}{R^{\frac{r-1}{r}}} \qquad or \qquad R^{\frac{r-1}{r}} = \frac{R_1^{-1/\gamma}}{R_1^{\frac{1-2r}{r}}}$$

$$R^{\frac{r-1}{r}} = R_1^{\frac{2(r-1)}{r}}$$

$$\text{Or } R = R_1^2 \qquad \text{i.e., } R_1 = \sqrt{R}$$

$$\therefore R_2 = \frac{R}{R_1} = \frac{R}{\sqrt{R}} = \sqrt{R}$$

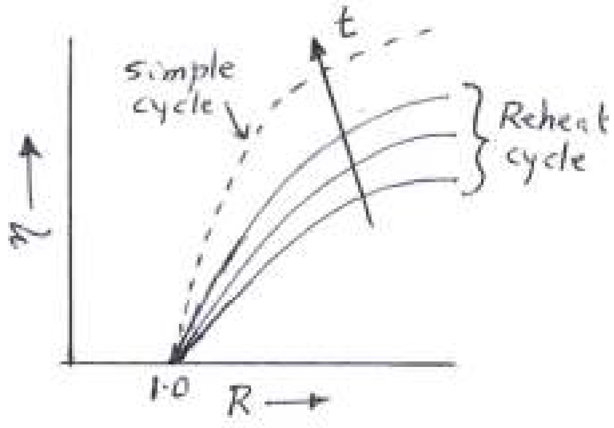
$\therefore$ For maximum work output, $R_1 = R_2 = \sqrt{R}$

Similarly if the cycle has ‘N’ stages of expansion, with reheating, then for maximum work output, pressure ratio for each = (compression pressure ratio)$^{1/N}$

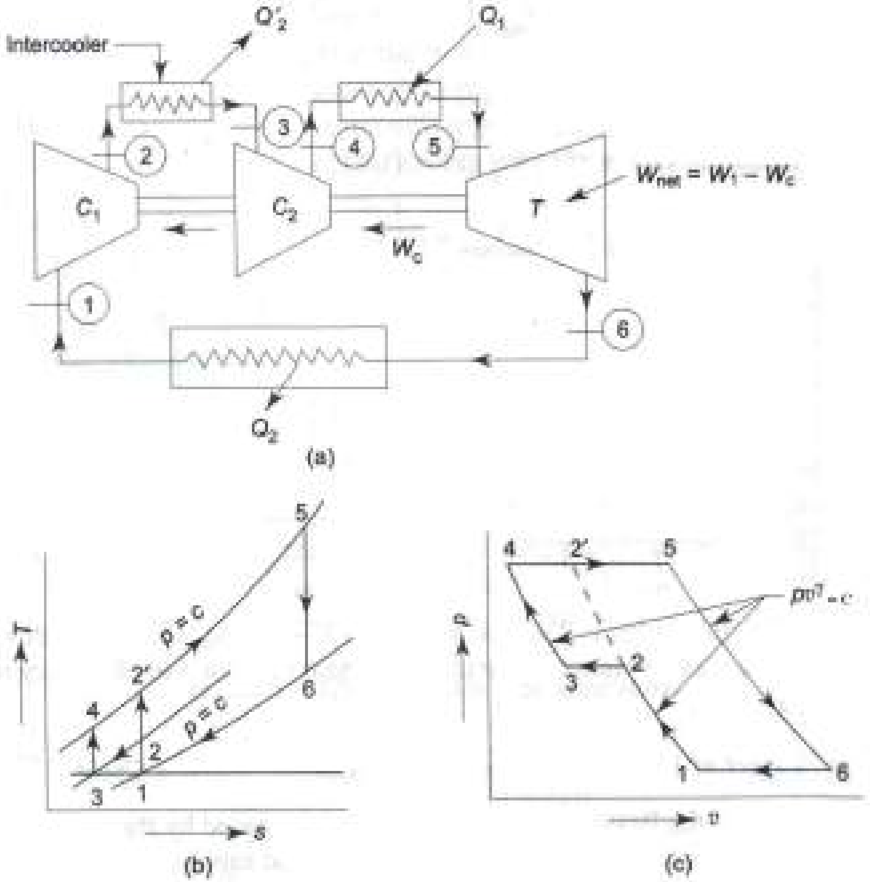
Advantages: By employing multistage expansion with reheating in between the stages, the net work output from the gas turbine cycle can be increased. This is illustrated on the T-S diagram shown for a 2-stage expansion with reheating in between the stages.

Disadvantages: But disadvantage of reheating is due to the fact that additional heat has to be supplied in order to reheat the air after each stage of expansion. This may result in a decrease in the thermal efficiency of the cycle. This is shown in figure below.

It can be seen that for a given value of t, the thermal η of the reheat cycle increases with increase in R and for a given value of R, the thermal η increases with increase in t. However, the thermal η of a reheat cycle will be less than that of a simple cycle for a given value of R.



b) Gas turbine cycle with multistage compression with inter cooling in between the stages:



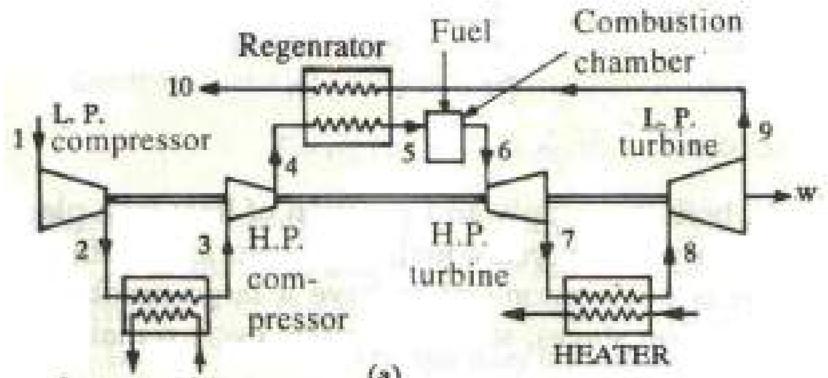
Effect of intercooling on Brayton cycle

Let $\frac{P_2}{P_1} = R_1$ $\quad P_4/P_3 = R_2$ $\quad P_5/P_6 = R$

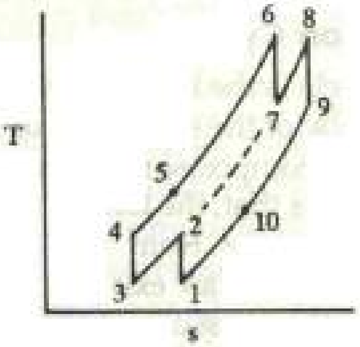
For maximum work output, $\quad R_1 = R_2 = \sqrt{R}$

The work output from a simple gas turbine cycle can be increased also by having multistage compression with inter cooling in between the stages. The effect of having two stage compression with inter cooling in between the stages is illustrated on the T-S diagram. It can be seen that, a higher work output has been achieved than that of simple cycle by an amount shown by the shaded area. The disadvantage of it is that more heat has to be supplied to heat the air than that is required for simple cycle. This may reduce the thermal $\eta$of the cycle.

c) Gas turbine cycle with two stage compression two-stage expansion and regenerator.



(a)



(b)

The thermal η of a gas turbine cycle may be improved by incorporating multistage compression with intercooling between the stages and multistage expansion with reheating between the turbines and also providing a regenerator. There is a definite saving of work due to multistage compression with intercooling arrangement between the stages. Similarly, the work output of the turbine is increased by multistage expansion with reheating. As a result, the net work of the plant increases.

The thermal efficiency of the cycle is given by $\eta_{th} = 1 - \frac{Q_L}{Q_H}$

$$= 1 - \frac{(h_{10} - h_1) + (h_2 - h_3)}{(h_6 - h_5) + (h_8 - h_7)}$$

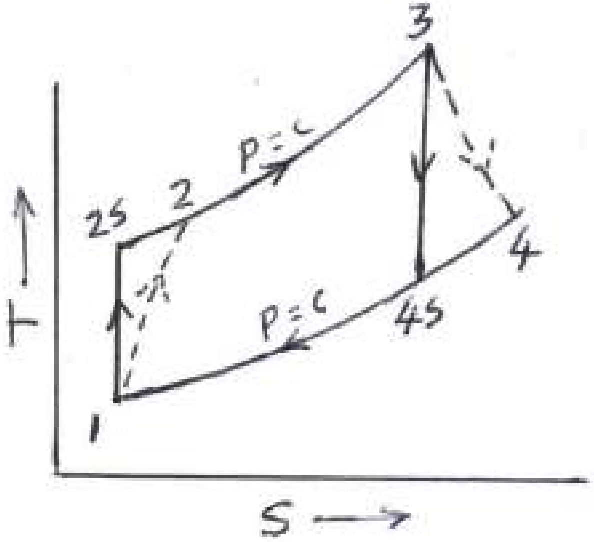
Deviation of Practical gas turbine cycle from ideal cycle:

1) The working substance will not be air through out the cycle. Air is compressed in compressor where as the products of combustion coming out of the combustion chamber is expanded in the turbine. The value of C_P and γ will be different for expansion and heating as compared to compression process.
   For compression, $C_P = 1.005$ kJ/kg-^{0}K, $\gamma = 1.4$
   For expansion, $C_P = 1.135$ kJ/kg-^{0}K, $\gamma = 1.33$

2) There will be pressure loss in the piping connecting the various components of the plant. $\therefore$ the pressure with which the products of combustion enters the turbine will be less than the

pressure with which air is coming out of the compressor i.e., the pressure ratio for expansion will be less than pressure ratio for compression.

3)



1-2s⇒ isentropic work of compression
1-2 ⇒ actual work of compression
3-4s ⇒ isentropic turbine work
3-4 ⇒ Actual turbine work

In a practical gas turbine cycle the compression and expansion processes are not isentropic but adiabatic with certain amount of frictional losses. The friction losses are accounted for by defining a parameter called isentropic efficiency.

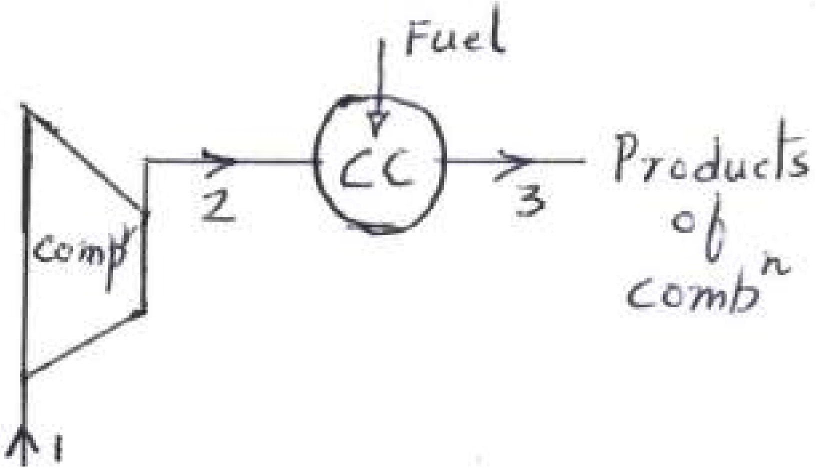
a) Isentropic η of compression (η_C)

$$\eta_C = \frac{Isentropic\ work\ of\ compression}{Actual\ work\ of\ compression} = \frac{T_{2S} - T_1}{T_2 - T_1}$$

b) Isentropic η of turbine (η_T)

$$\eta_T = \frac{Actual\ turbine\ work}{Isentropic\ turbine\ work} = \frac{T_3 - T_4}{T_3 - T_{4S}}$$

To determination the air-fuel ratio:



Let m_a = mass of air entering the combustion chamber
m_f = mass of fuel entering the combustion chamber
CV = Calorific value of fuel

Applying SFEE to combustion chamber, we get,

$$m_a h_2 + m_f CV = (m_a + m_f) h_3$$

÷by m_f $$\frac{m_a}{m_f} h_2 + CV = \left(\frac{m_a}{m_f} + 1\right) h_3$$

$$\cong \frac{m_a}{m_f} h_3 \quad as \quad \frac{m_a}{m_f} >>>>> 1$$

$$\therefore \frac{m_a}{m_f}\left(h_3 - h_2\right) = CV$$

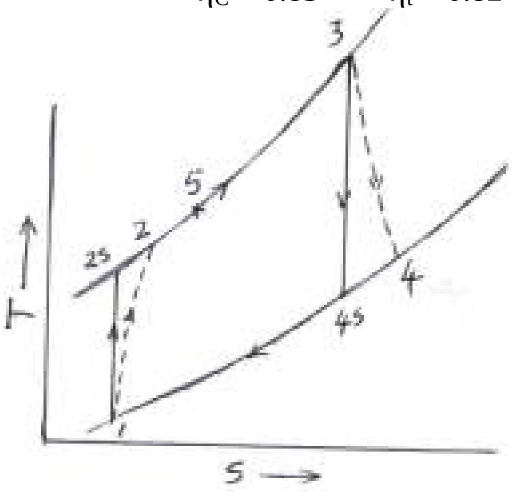
$$or \quad \frac{m_a}{m_f} = \frac{CV}{\left(h_3 - h_2\right)} \qquad i.e., \quad \frac{m_a}{m_f} = \frac{CV}{C_P\left(T_3 - T_2\right)}$$

Problems:

1. **In a G.T. installation, the air is taken in at 1 bar and 15^0C and compressed to 4 bar. The isentropic η of turbine and the compressor are 82% and 85% respectively. Determine (i) compression work, (ii) Turbine work, (iii) work ratio, (iv) Th. η.**

   What would be the improvement in the th. η if a regenerator with 75% effectiveness is incorporated in the cycle. Assume the maximum cycle temperature to be 825^0K.

Solution: P_1 = 1 bar $T_1 = 288^0$K P_2 = 4 bar $T_3 = 825^0$K
η_C = 0.85 η_t = 0.82



Process 1-2 is isentropic i.e., $\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}$

$$\therefore T_{2s} = 288\,(4)^{\frac{0.4}{1.4}} = 428.14^0\,K$$

But $\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1}$ $i.e., 0.85 = \frac{428.14 - 288}{T_2 - 288}$ $\therefore T_2 = 452.87^0\,K$

Process 3-4 is isentropic $i.e.,$ $\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}}$ $\therefore T_{4s} = 825\left(\frac{1}{4}\right)^{\frac{0.4}{1.4}} = 554.96$

But $\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}}$ $i.e., 0.82 = \frac{825 - T_4}{825 - 554.96}$ $\therefore T_4 = 603.57^0\,K$

(i) Compressor work, $W_C = C_P\,(T_2 - T_1)$

$= 1.005\,(452.87 - 288) = 165.69$ kJ/kg

(ii) Turbine work, $W_t = C_P\,(T_3 - T_4)$

$= 1.005\,(825 - 603.57) = 222.54$ kJ/kg

(iii) Work ratio $= W_R = \frac{Net\ work\ output}{Turbine\ work} = \frac{W_T - W_C}{W_T} = 0.255$

(iv) Th. η, $\eta_{th} = \frac{W_{net}}{Q_H} = \frac{222.54 - 165.69}{C_P(825 - 452.87)} = \frac{56.85}{373.99}$

$= 15.2\%$

we have effectiveness $= \frac{T_5 - T_2}{T_4 - T_2} = 0.75 = \frac{T_5 - 452.87}{603.57 - 452.87}$

$\therefore T_5 = 565.89^0 K$

$\therefore$ Heat supplied, $Q_H^1 = Q_{5-3} = C_P(T_3 - T_5)$

$= 1.005\,(825 - 565.89)$

$= 260.4$ kJ/kg

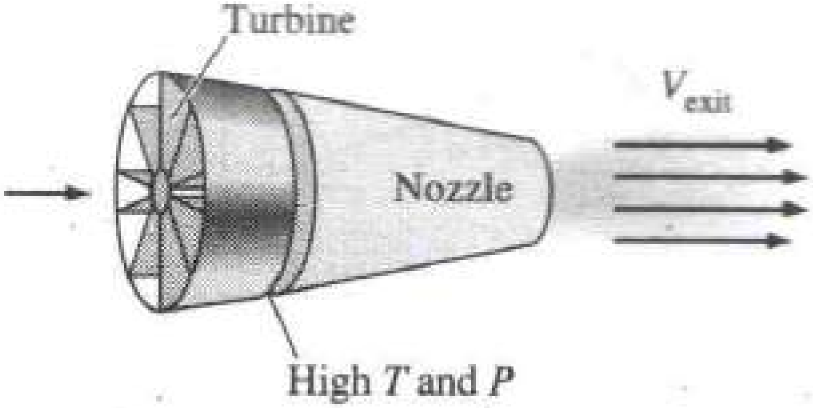
$$\therefore \eta_{th} = \frac{W_T - W_C}{Q_H^{\ 1}} = \frac{56.85}{260.4} = 0.218$$

$\therefore$ Improvement in η_{th} due to regenerator $= \frac{0.218 - 0.152}{0.152}$

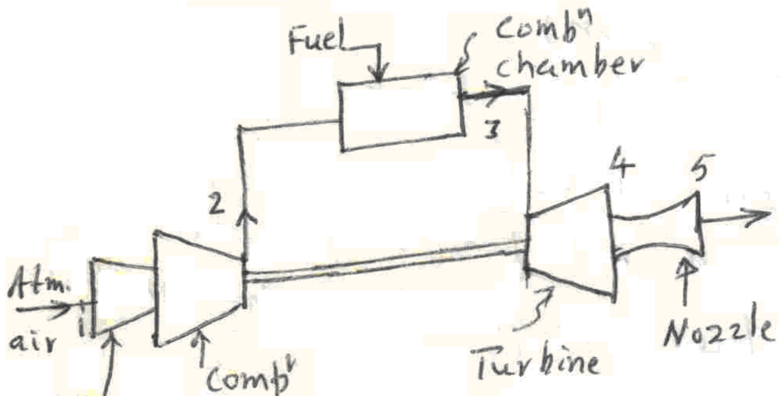
$= 0.436$ i.e., 43.6%

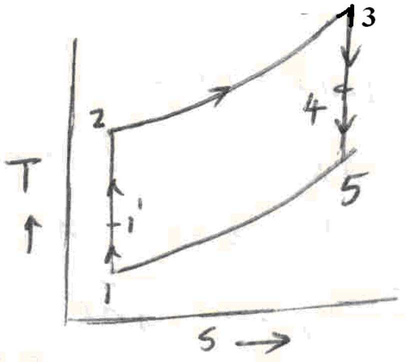
Ideal Jet-Propulsion Cycles:

Gas turbine engines are widely used to power air-crafts because they are light and compact and have a high power-to-weight ratio. Air craft gas turbine operates on an open cycle called a jet-propulsion cycle. The ideal jet propulsion cycle differs from the simple ideal Brayton cycle in that the gases are not expanded to the ambient pressure in the turbine. Instead, they are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor and the auxiliary equipment, such as a small generator and hydraulic pumps. That is, the net work output of a jet propulsion cycle is zero. The gases that exit the turbine at relatively high pressure are subsequently accelerated in a nozzle to provide the thrust to propel the air craft. Also, air craft gas turbine operate at higher pressure ratios (typically between 10-25), and the fluid passes through a diffuser first, where it is decelerated and its pressure is increased before it enters the compressor.



Air standard Jet Propulsion cycle:





An air standard Brayton or Joule cycle is the basic cycle for jet propulsion. The jet engine consists practically of the same type of components as in gas turbine plants, namely a compressor, a combustion chamber and a turbine. The only difference is that, the former also consists of an inlet diffuser, where the air entering from the atmosphere is decelerated and slightly compressed, and an exit nozzle, where the products of combustion expand to the pressure of the surroundings with increase in relative velocity. Such a plant and the corresponding T-S diagram are shown in Fig. In this plant, the work output of the turbine is just sufficient to drive the compressor.

Air enters the diffuser from the atmosphere, is slightly compressed from state 1 to state 1^1. It then enters the compressor, where it is further compressed to state 2. The compressed air then flows into the combustion chamber where it burns the fuel at constant pressure from state 2 to state 3 and the products of combustion then expand in the turbine from state 3 to state 4 developing power which is just sufficient to drive the compressor. Further expansion of the burnt gases takes place in the exit nozzle at state 5, after which the gases make an exit into the atmosphere with a very high velocity. The momentum of the exhaust gases flowing at high velocity from the nozzle result in a thrust upon the aircraft on which the engine is installed. In an actual jet propulsion plant, there is a slight pressure drop in the combustion chamber and the processes of combustion and expansion are not strictly reversible adiabatic. The thrust of a jet plane is the propulsive force i.e.,

$F = \dot{m}_a(\bar{v}_5 - \bar{v}_1) + \dot{m}_f \bar{v}_5$ where $\dot{m}_a$ = mass flow rate of air,

$\bar{v}_1$ & $\bar{v}_5$ = inlet and exit velocity of the fluid

$\dot{m}_f$ = mass flow rate of fuel in the combustion chamber

but since $\dot{m}_f$ is very small, $\dot{m}_f \bar{v}_5$ term is neglected.

$$\therefore F = \dot{m}_a(\bar{v}_5 - \bar{v}_1)$$

Turbo jet:

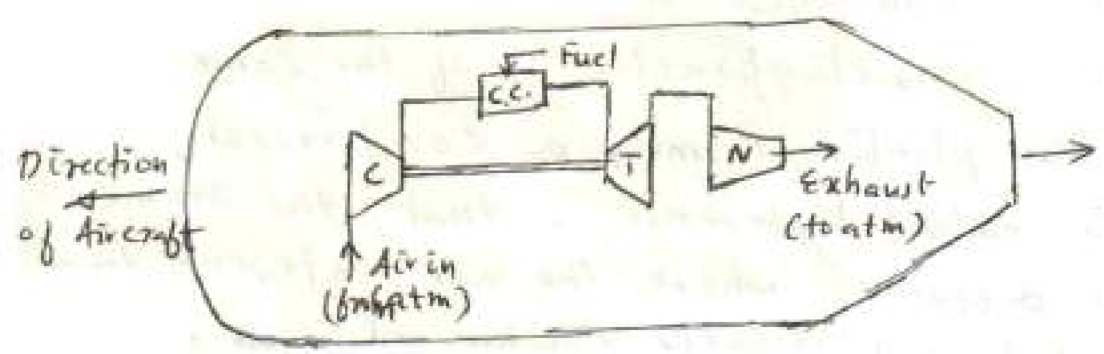


Figure shows a turbo jet unit. Power produced by the turbine is just sufficient to drive the compressor. The exhaust gases from the turbine which are at a higher pressure than atmosphere are expanded in the nozzle and a very high velocity jet is produced which provides a forward motion to the air-craft by the jet reaction (Newton's third law of motion).

At higher speeds the turbo jet gives higher propulsion efficiency. The turbo-jets are most suited to the air-crafts traveling above 800 km/hr. The overall η of a turbo jet is the product of the thermal η of the gas turbine plant and the propulsive η of the jet (nozzle).

Turbo-Propellers:

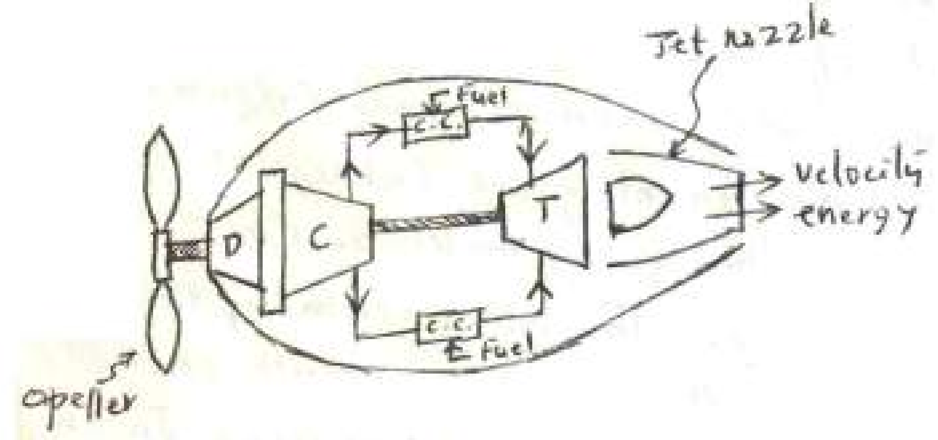


Figure shows a turbo-propeller system used in air-crafts. Here expansion takes place partly in turbine (80%) and partly (20%) in the nozzle. The power developed by the turbine is consumed in running the compressor and the propeller. The propeller and jet produced by the nozzle give forward motion to the aircraft.

It is having an added advantage over turbojet i.e., low specific weight and simplicity in design and propeller i.e., high power for take-off and high propulsion η at speeds below 600 km/hr.

Its overall η is improved by providing the diffuser before compressor, which increases the pressure rise. This pressure rise takes place due to conversion of kinetic energy of the incoming air (equal to air-craft velocity) into pressure energy by the diffuser. This type of compression is called 'ram effect'.

Ram jet: Ram jet engines have the capacity to fly at supersonic speeds. A diffuser increases the pressure of incoming air which is adequate to self ignite the fuel. In a ram jet engine the temperature of the rammed air is always above the self ignition temperature of the fuel employed. The ram-jet engine consists of a diffuser, combustion chamber and nozzle. The air enters the ram-jet plant with supersonic speed and is slowed down to sonic velocity in the supersonic diffuser, consequently the pressure suddenly increases to the formation of shock wave. The pressure of the air is further increased in the subsonic diffuser, increasing the temperature of air above the ignition temperature. The burning of the fuel takes place in the combustion chamber with the help of flame stabilization. The high pressure and high temperature gases pass through the nozzle where the pressure energy is converted to kinetic energy. The high velocity gases leaving the nozzle is a source of forward thrust to the ram-jet.

Advantages:

i) No moving parts
ii) Wide variety of fuels may be used
iii) Light in weight

The major short coming of ram-jet engine is that it cannot be started of its own. It has to be accelerated to certain flight velocity by some launching device. It is always equipped with a small turbo-jet which starts the ram-jet.

Rocket Propulsion: Similar to jet propulsion, the thrust required for rocket propulsion is produced by the high velocity jet of gases passing through the nozzle. But the main difference is that in case of jet propulsion the oxygen required for combustion is taken from the atmosphere and fuel is stored whereas for rocket engine, the fuel and oxidizer both are contained in a propelling body and as such it can function in vacuum also.

Rockets are classified as follows:

I According to the type of propellants

i) Solid propellant rocket
ii) Liquid propellant rocket

II According to the number of motors

i) Single-stage rocket (consists of one rocket motor)
ii) Multi-stage rocket (consists more than one rocket motor)

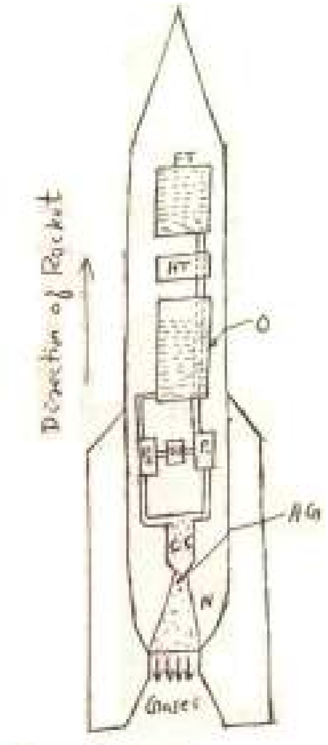


Fig :- Rocket

FT = Fuel tank
HT = Hydrogen peroxide tank
O = Oxidizer tank
ST = Steam turbine
P_1, P_2 = Pumps
C.C = Combustion chamber
HG = Hot gases
N = Nozzle

Figure shows a simple type single stage liquid propellant (the fuel and the oxidizer are commonly known as propellants) rocket. It consists of a fuel tank FT, an oxidizer tank O, two pumps P_1, P_2, a steam turbine ST and a combustion chamber C.C. The fuel tank contains alcohol and oxidizer tank contains liquid oxygen. The fuel and the oxidizer are supplied by the pumps to the combustion chamber where the fuel is ignited by electrical means. The pumps are driven with the help of steam turbine. Here the steam is produced by mixing a very concentrated hydrogen-peroxide with potassium permanganate. The products of combustion are discharged from the combustion chamber through the nozzle N. So the rocket moves in the opposite direction.

In some modified forms, this type of rockets may be used in missiles.

Uses:

1. Long range artillery
2. Signaling and fire work display
3. Jet assisted take-off
4. For satellites
5. For space ships
6. Research

Effect of Boiler Pressure (Using Molliar Diagram i.e., h-s diagram)

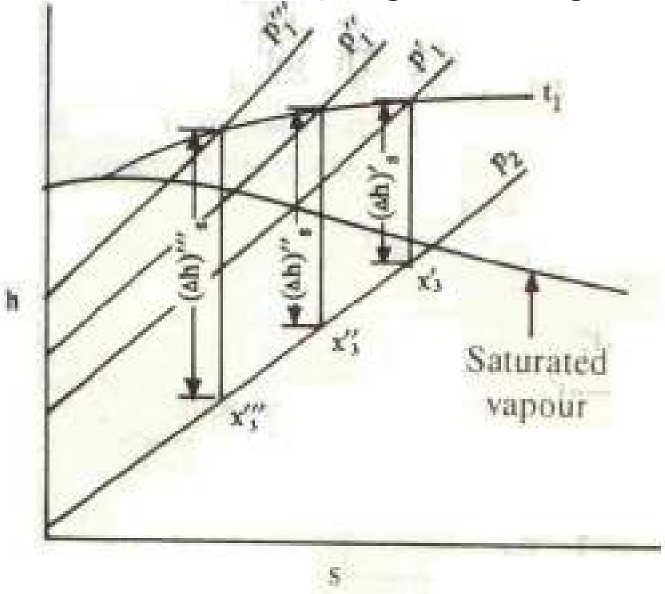


Fig. 13.5 Effect of boiler pressure

We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P << W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_S}{(h_2 - h_1)}$$

i.e., Rankine cycle η depends on h_2, h_1 and Δh_S. From figure as $P_1''' > P_1'' > P_1'$ for the fixed maximum temperature of the steam t_1 and condenser pressure P_2, Isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop $(\Delta h)_S$ increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., $x_3''' < x_3'' < x_3'$, which leads to serious wear of the turbine blades.

Effect of Super Heating (Using Molliar Diagram i.e., h-s diagram)

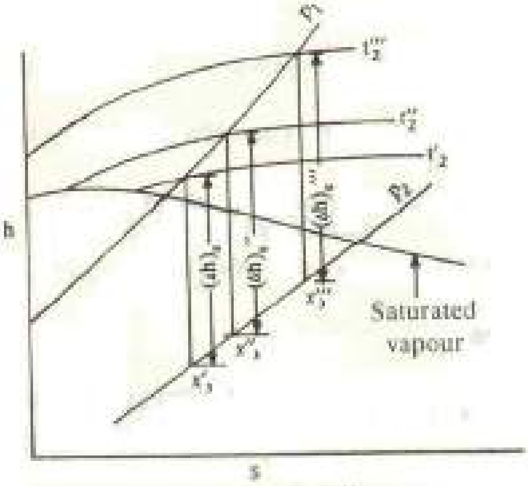
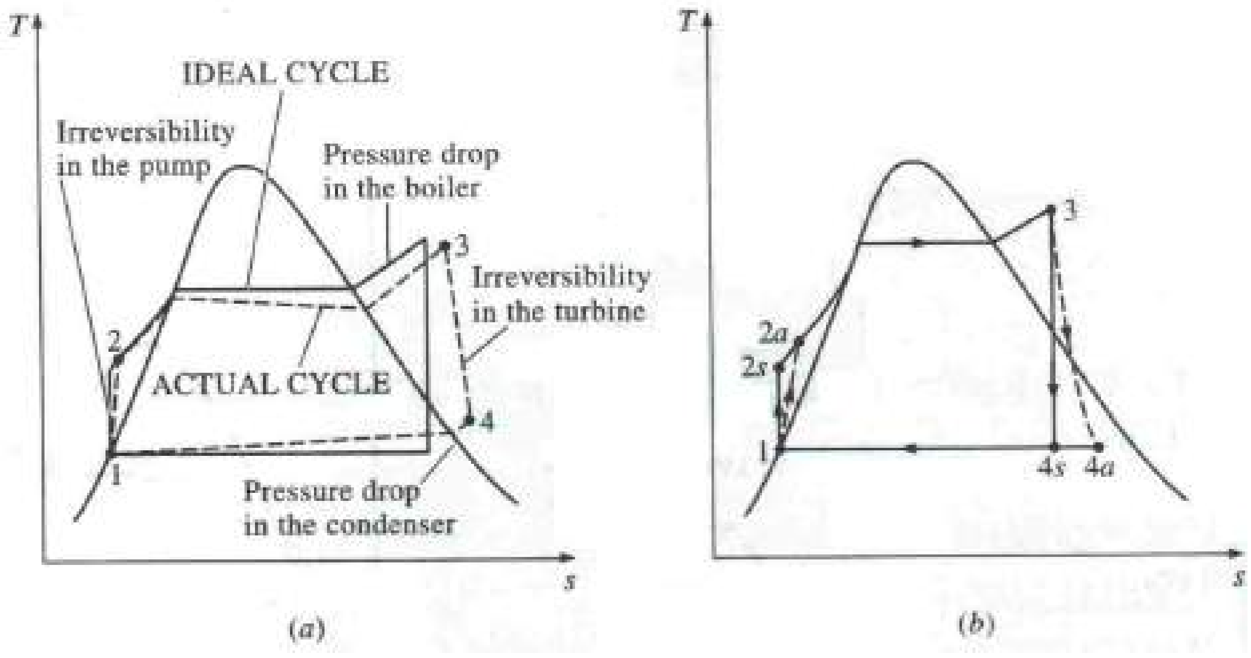


Fig. Effect of superheating

The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t_1. This can be seen in figure where $t_1''' > t_1'' > t_1'$, but $x_3' < x_3'' < x_3'''$. It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Deviation of Actual Vapour Power cycles from Ideal cycle



(a) (b)

The actual Vapour power cycle differs from the ideal Rankine cycle, as shown in figure, as a result of irreversibilities in various components mainly because of fluid friction and heat loss to the surroundings.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a lower pressure. Also the pressure at the turbine inlet is lower than that at the boiler exit due to pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate these pressure drops, the water must be pumped to sufficiently higher pressure which requires the larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

$$\eta_P = \frac{W_S}{W_a} = \frac{h_{1S} - h_4}{h_1 - h_4}$$

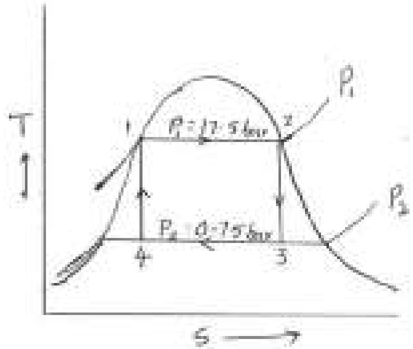
And $\eta_t = \dfrac{W_a}{W_S} = \dfrac{h_2 - h_3}{h_2 - h_{3S}}$

Numerical Problems:

1. **Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: P_1 = 17.5 bar P_2 = 0.75 bar η_{Carnot} = ? $\eta_{Rankine}$ = ?

a) Carnot cycle: At pressure 17.5 bar from steam tables,



P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For P = 17.5 bar, using linear interpolation

For t_S, $204.3 + \frac{207.11 - 204.3}{1} x0.5 = 205.71^0 C$

$= 478.71$ K

Similarly, $h_f = 878.15$ kJ/kg $h_{fg} = 1915.95$ kJ/kg $h_g = 2794.1$ kJ/kg
$S_f = 2.3844$ kJ/kg^{0}K $S_{fg} = 4.0011$ kJ/kg^{0}K $S_g = 6.3855$ kJ/kg K

Also at pressure 0.75 bar from steam tables

P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.4352
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.4804

$\therefore$ For 0.75 bar, using linear interpolation,
$t_S = 91.74^0$C $h_f = 384.25$ $h_{fg} = 2278.65$ $h_g = 2662.95$
$S_f = 1.2126$ $S_{fg} = 6.2453$ $S_g = 7.4578$

The Carnot cycle η, $\eta_C = \frac{T_1 - T_2}{T_1} = \frac{478.71 - 364.74}{478.71} = 0.2381$

Steam rate or SSC = $\frac{1}{\oint \delta W} = \frac{1}{W_T - W_P}$

Since the expansion work is isentropic, $S_2 = S_3$

But $S_2 = S_g = 6.3855$ and $S_3 = S_{f3} + x_3 S_{fg3}$

i.e., $6.3855 = 1.2126 + x_3 (6.2453)$ $\therefore x_3 = 0.828$

$\therefore$ Enthalpy at state 3, $h_3 = h_{f3} + x_3 h_{fg3}$
$= 384.25 + 0.828 (2278.65) = 2271.63$ kJ/kg

$\therefore$ Turbine work or expansion work or positive work $= h_2 - h_3$
$= 2794.1 - 2271.63 = 522.47$ kJ/kg

Again since the compression process is isentropic
i.e., $S_4 = S_1 = S_{f1} = 2.3844$

Hence $2.3844 = S_{f4} + x_4 S_{fg4}$
$= 1.2126 + x_4 (6.2453)$ $\therefore x_4 = 0.188$

$\therefore$ Enthalpy at state 4 is $h_4 = h_{f4} + x_4 h_{fg4}$

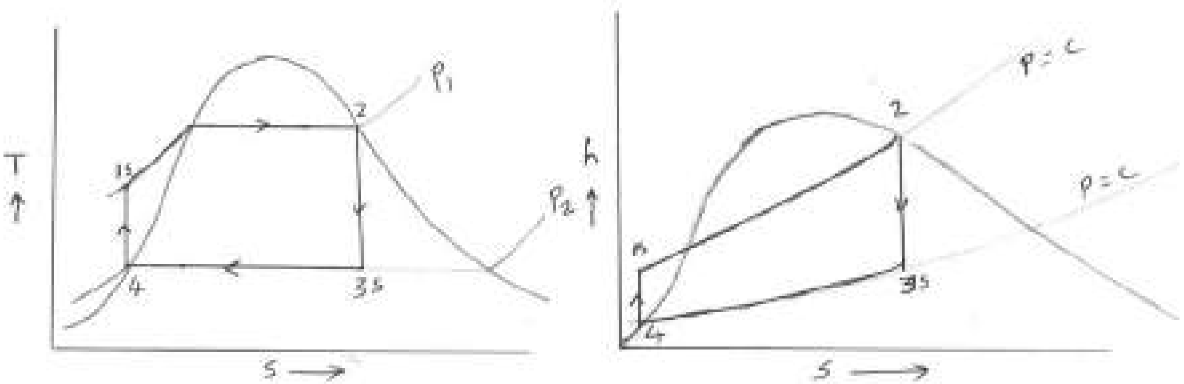
= 384.25 + 0.188 (2278.65)
= 811.79 kJ/kg

$\therefore$ Compression work, = $h_1 - h_4$ = 878.15 – 811.79
W_P = 66.36 kJ/kg

$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192\, x\, 10^{-3} kg / kJ$$

$$\text{work ratio} = r_w = \frac{\oint \delta w}{+ve\, work} = \frac{W_T - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

b) Rankine cycle:



$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process, work W = $-v\int dp$

$\therefore W_P = h_{1S} - h_4 = v_{fP2} (P_1 - P_2)$
= 0.001037 (17.5 – 0.75) x 10^5 x (1/1000)
= 1.737 kJ/kg

$\therefore h_{1S}$ = 1.737 + 384.25 = 385.99 kJ/kg

Hence, turbine work = $W_T = h_2 - h_3$ = 522.47kJ/kg
Heat supplied = $Q_H = h_2 - h_{1S}$ = 2.794.1 – 385.99 = 2408.11 kJ/kg

$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$

$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204\, x\, 10^{-3} kg / kJ$$

Work ratio, $r_w = \dfrac{522.47 - 1737}{522.47} = 0.9967$

2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{W_P}{0.85} = \frac{1.737}{0.85} = 2.0435 kJ/kg$$

$W_T = \eta_T W_T = 0.85\ (522.47) = 444.09$ kJ/kg

$\therefore W_{net} = W_T - W_P = 442.06$ kJ/kg

$\therefore$ % reduction in work output = $\dfrac{520.73 - 442.06}{520.73} = 15.11\%$

$W_P = h_{1S} - h_4$ $\therefore h_{1S} = 2.0435 + 384.25 = 386.29$ kJ/kg

$\therefore Q_H - h_2 - h_{1S} = 2794.1 - 386.29 = 2407.81$ kJ/kg

$\therefore \eta_{cycle} = \dfrac{442.06}{2407.81} = 0.1836$

$\therefore$ % reduction in cycle efficiency $= \dfrac{0.2162 - 0.1836}{0.2162} = 15.08\%$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows

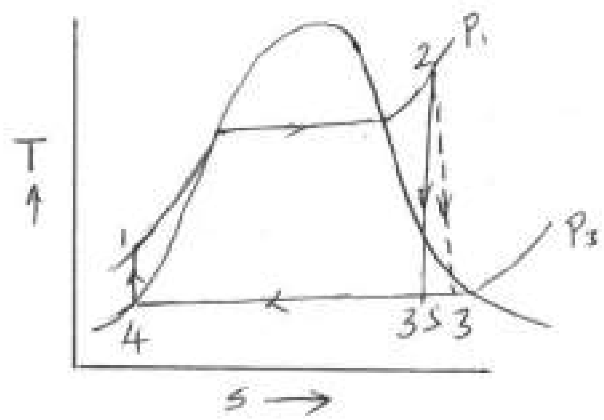
Since steam is dry saturated at state 2, locate this state at the pressure P_2 = 17.5 bar on the saturation line and read the enthalpy at this state. This will give the value of h_2.

As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, P = 0.75 bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3.

The value of h_4 can be found from the steam tables at pressure, P = 0.75 bar, as $h_4 = h_{f4}$. After finding the values of h_2, h_3 and h_4, apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.

3. **Steam enters the turbine of a steam power plant, operating on Rankine cycle, at 10 bar, 300^0C. The condenser pressure is 0.1 bar. Steam leaving the turbine is 90% dry. Calculate the adiabatic efficiency of the turbine and also the cycle η, neglecting pump work.**

Solution:



P_1 = 10 bar $t_2 = 300^0$C P_3 = 0.1 bar

x_3 = 0.9 η_t = ? η_{cycle} = ?   Neglect W_P

From superheated steam tables,

For P_2 = 10 bar and $t_2 = 300^0$C, h_2 = 3052.1 kJ/kg, s_2 = 7.1251 kJ/kg

From table A – 1, For P_3 = 0.1 bar

$t_S = 45.83^0$C h_f = 191.8 h_{fg} = 2392.9

S_f = 0.6493 S_{fg} = 7.5018

Since x_3 = 0.9, $h_3 = h_{f4} + x_3\, h_{fg3}$

= 191.8 +0.9 (2392.9)

= 2345.4 kJ/kg

Also, since process 2-3s is isentropic, $S_2 = S_{3S}$

i.e., 7.1251 = $S_{fg4} + x_{3S}\, S_{fg3}$

= 0.6493 + x_{3S} (7.5018)

$\therefore x_{3S}$ = 0.863

$\therefore h_{3S}$ = 191.8 + 0.863 (2392.9) = 2257.43 kJ/kg

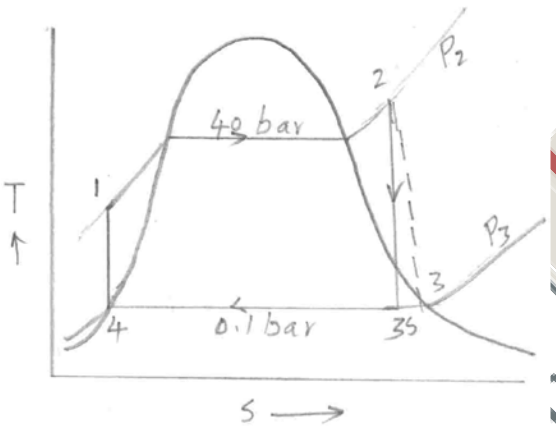
$$\therefore \textit{Turbine efficiency}, \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} = \frac{3052.1 - 2345.4}{3052.1 - 2257.43} = 0.89$$

$$\eta_{cycle} = \frac{W_T}{Q_H} = \frac{h_2 - h_3}{h_2 - h_1} \quad but \quad h_1 = 191.8 \text{ kJ/kg}$$

$$= \frac{3052.1 - 2345.4}{3052.1 - 191.8} = 0.25 \quad i.e., \quad 25\%$$

4. **A 40 mW steam plant working on Rankine cycle operates between boiler pressure of 4 MPa and condenser pressure of 10 KPa. The steam leaves the boiler and enters the steam turbine at 400^{0}C. The isentropic η of the steam turbine is 85%. Determine (i) the cycle η (ii) the quality of steam from the turbine and (iii) the steam flow rate in kg per hour. Consider pump work.**

Solution:



P_2 = 4 MPa = 40 bar P_3 = 10 KPa = 0.1 bar

P = 40000kW $t_2 = 400^0$C $\eta_t = 0.85$ η_{cycle} = ? x_3 = ?

$\dot{m} = ?$

$$h_2 = h\Big|_{40\,bar,\,400^0 C} = 3215.7 kJ/kg \text{ and } s_2 = 6.7733 \text{ kJ/kg-K}$$

$$h_4 = h_f\Big|_{0.1 bar} = 191.8 kJ/kg$$

Process 2-3s is isentropic i.e., $S_2 = S_{3S}$

$$6.7733 = 0.6493 + x_{3S}(7.5018)$$

$$\therefore x_{3S} = 0.816$$

$\therefore h_{3S} = h_{f3} + x_{3S}\, h_{fg3} = 191.8 + 0.816\,(2392.9)$

$= 2145.2$ kJ/kg

$$But \quad \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} \quad i.e., \quad 0.85 = \frac{3215.7 - h_3}{3215.7 - 2145.2}$$

$\therefore h_3 = 2305.8$ kJ/kg

$\therefore W_T = h_2 - h_3 = 3215.7 - 2305.8 = 909.9$ kJ/kg

$W_P = v\int dP \quad = 0.0010102\,(40 - 0.1)\, 10^5/10^2$

$= 4.031$ kJ/kg

$= h_1 - h_4 \qquad \therefore h_1 = 195.8$ kJ/kg

$$(i)\ \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{909.9 - 4.031}{(3215.7 - 195.8)} = 29.9\%$$

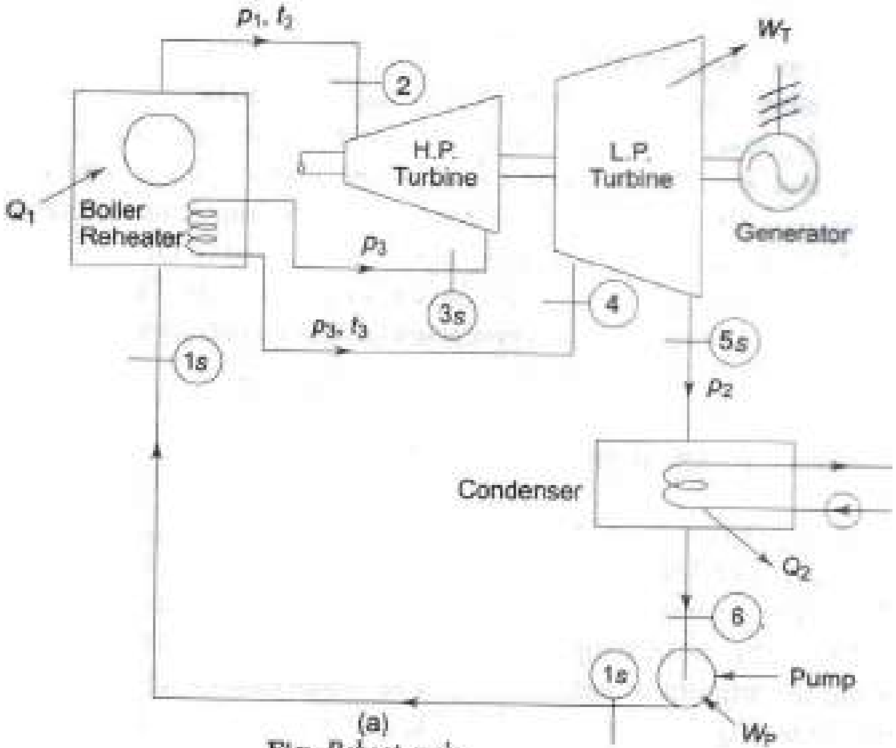
(ii) $x_3 = ?$ we have $2305.8 = 191.8 + x_3\,(2392.9)$ $\therefore x_3 = 0.88$

(iii) $P = \dot{m}\, W_{net}$ i.e., $40000 = \dot{m}\,(905.87)$

$\therefore \dot{m} = 44.2$ kg/s

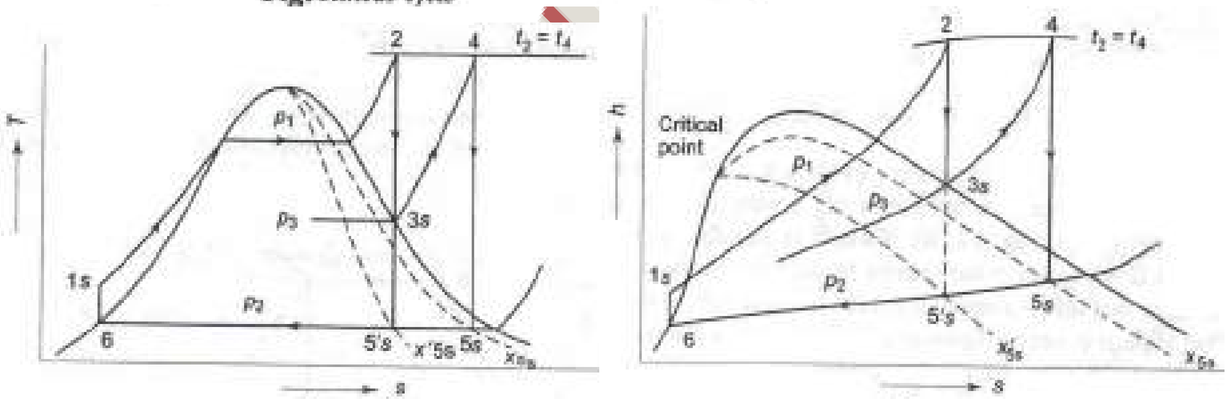
$= 159120$ kg/hr

Ideal Reheat cycle: We know that, the efficiency of the Rankine cycle could be increased by increasing steam pressure in the boiler and superheating the steam. But this increases the moisture content of the steam in the lower pressure stages in the turbine, which may lead to erosion of the turbine blade. ∴The reheat cycle has been developed to take advantage of the increased pressure of the boiler, avoiding the excessive moisture of the steam in the low pressure stages. In the reheat cycle, steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion.



(a)

Fig. *Reheat cycle*



(b) (c)

In the reheat cycle the expansion of steam from the initial state (2) to the condenser pressure is carried out in two or more steps, depending upon the number of reheats used.

In the first step, steam expands in HP turbine from state 2 to approximate the saturated vapour line (process 2-3s). The steam is then reheated (or resuperheated) at constant pressure in the boiler (or in a reheater) process 3s-4 and the remaining expansion process 4s-5 is carried out in the LP turbine.

Note: 1) To protect the reheater tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating, because in that case the moisture particles in steam while evaporating would leave behind solid deposits in the form of scale which is difficult to remove. Also a low reheat pressure may bring down T_{m1} and hence cycle η. Again a high reheat pressure increases the moisture content at turbine exhaust. Thus reheat pressure is optimized. Optimum reheat pressure is about 0.2 to 0.25 of initial pressure.

We have for 1 kg of steam

$Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S})$; $Q_L = h_{5S} - h_6$
$W_T = (h_2 - h_{3S}) + (h_4 - h_{5S})$; $W_P = h_{1S} - h_6$

$$\therefore \quad \eta_R = \frac{W_T - W_P}{Q_H} \; ;$$

Steam rate $= \dfrac{3600}{(W_T - W_P)} kg / kWh$

Since higher reheat pressure is used, W_P work is appreciable.

2) In practice, the use of reheat gives a marginal increase in cycle η, but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit. The quality improves from x_{5^1S} to x_{5S} by the use of reheat.

Ideal Regenerative cycle: The mean temperature of heat addition can also be increased by decreasing the amount of heat added at low temperatures. In a simple Rankine cycle (saturated steam entering the turbine), a considerable part of the total heat supplied is in the liquid phase when heating up water from 1 to 1^1, at a temperature lower than T_2, the maximum temperature of the cycle. For maximum η, all heat should be supplied at T_2, and feed water should enter the boiler at 1^1. This may be accomplished in what is known as an ideal regenerative cycle as shown in figures (a) and (b).

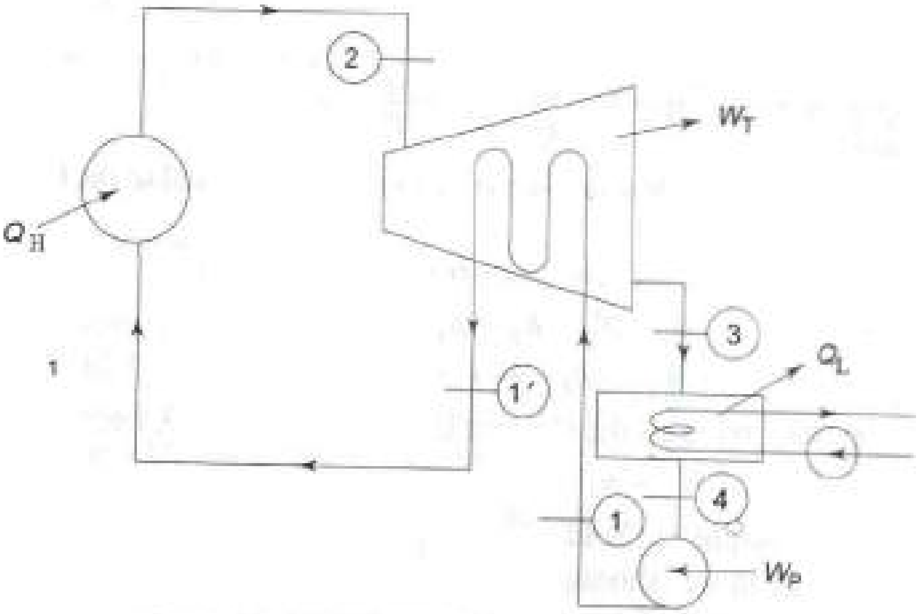


Fig. *Ideal regenerative cycle*

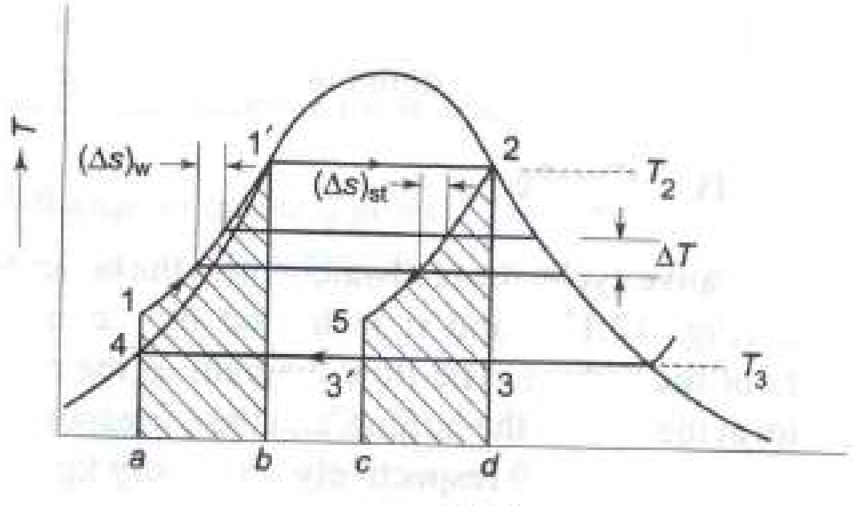


Fig. *Ideal regenerative cycle on T-s plot*

$$\Delta T(\text{water}) = -\Delta T\,(\text{steam})$$

The unique feature of the ideal regenerative cycle is that the condensate, after leaving the pump circulates around the turbine casing, counter-flow to the direction of vapour flow in the turbine. Thus it is possible to transfer heat from the vapour as it flows through the turbine to the liquid flowing around the turbine.

Let us assume that this is a reversible heat transfer i.e., at each point, the temperature of the vapour is only infinitesimally higher than the temperature of the liquid. $\therefore$The process 2-3^1 represents reversible expansion of steam in the turbine with reversible heat rejection. i.e., for any small step in the process of heating the water $\Delta T_{\text{(water)}} = -\Delta T_{\text{(steam)}}$ and $(\Delta S)_{\text{water}} = (\Delta S)_{\text{steam}}$. Then the slopes of lines 2-3^1 and 1^1-4 will be identical at every temperature and the lines will be identical in contour. Areas 1-1^1-b-a-1 and 3^1-2-d-c-3^1 are not only equal but congruous. $\therefore$, all heat added from external source (Q_H) is at constant temperature T_2 and all heat rejected (Q_L) is at constant temperature T_3, both being reversible.

Then $\quad Q_H = h_2 - h_1^1 = T_2 (S_2 - S_1^1)$

$\qquad Q_L = h_3^1 - h_4 = T_3 (S_3^1 - S_4)$

Since $S_1^1 - S_4 = S_2 - S_3^1$ or $S_2 - S_1^1 = S_3^1 - S_4$

$\therefore \quad \eta_{\mathrm{Re}\,g} = 1 - \dfrac{Q_L}{Q_H} = 1 - \dfrac{T_3}{T_2}$ i.e., the η of ideal regenerative cycle is thus equal to the Carnot cycle η.

Writing SFEE to turbine,

$h_2 + h_1 = W_T + h_1^1 + h_3^1$

i.e., $W_T = (h_2 - h_3^1) - (h_1^1 - h_1)$

or $W_T = (h_2 - h_3^1) - (h_1^1 - h_1)$ --- (1)

and the W_P is same as simple rankine cycle i.e., $W_P = (h_1 - h_4)$

$\therefore$The net work output of the ideal regenerative cycle is less and hence its steam rate will be more. Although it is more efficient when compared to rankine cycle, this cycle is not practicable for the following reasons.

1) Reversible heat transfer cannot be obtained in finite time.
2) Heat exchanger in the turbine is mechanically impracticable.
3) The moisture content of the steam in the turbine is high.

Regenerative cycle:

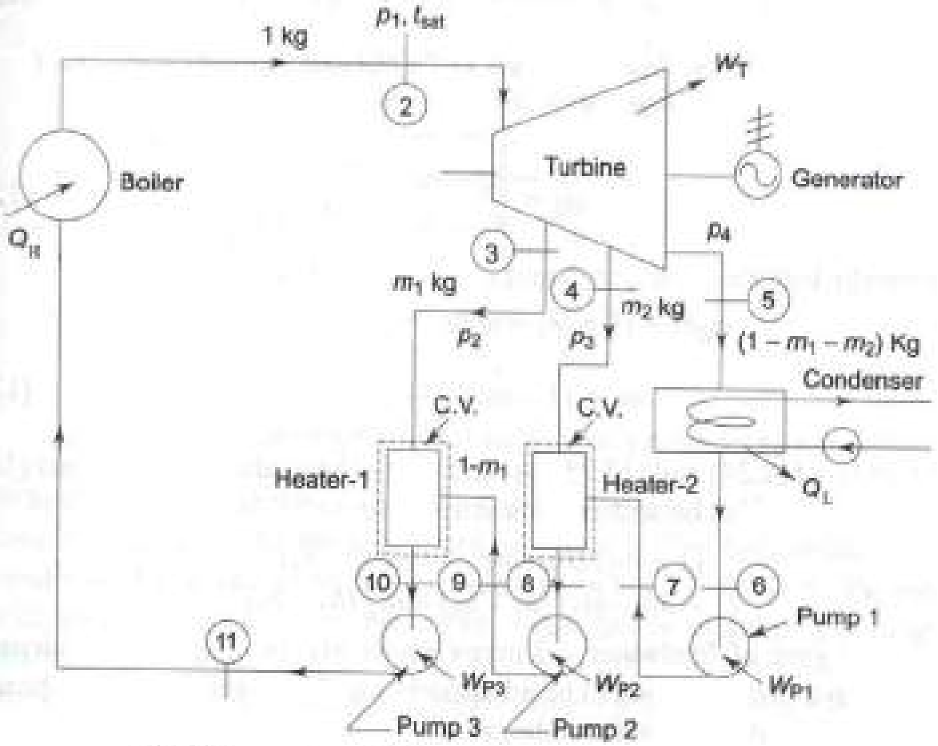
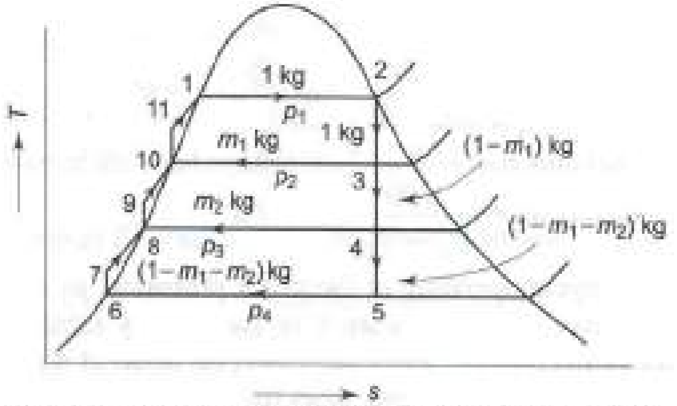
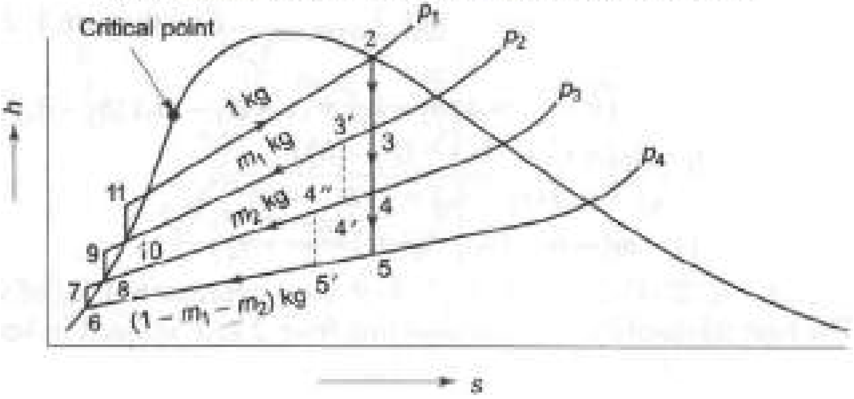


Fig. *Regenerative cycle flow diagram with two feedwater heaters*



(a) Regenerative cycle on T-s plot with decreasing mass of fluid



In a practical regenerative cycle, the feed water enters the boiler at a temperature between 1 and 1^1 (previous article figure), and it is heated by steam extracted from intermediate stages of the turbine. The flow diagram of the regenerative cycle with saturated steam at the inlet to the turbine and the corresponding T-S diagram are shown in figure.

For every kg of steam entering the turbine, let m_1 kg steam be extracted from an intermediate stage of the turbine where the pressure is P_2, and it is used to heat up feed water [$(1 - m_1)$ kg at state 9] by mixing in heater (1). The remaining $(1-m_1)$ kg of steam then expands in the turbine from pressure P_2 (state 3) to pressure P_3 (state 4) when m_2 kg of steam is extracted for heating feed water in heater (2). So $(1 - m_1 - m_2)$kg of steam then expands in the remaining stages of the turbine to pressure P_4, gets condensed into water in the condenser, and then pumped to heater (2), where it mixes with m_2 kg of steam extracted at pressure P_3. Then $(1-m_1)$ kg of water is pumped to heater (1) where it mixes with m_1 kg of steam extracted at pressure P_2. The resulting 1kg of steam is then pumped to the boiler where heat from an external source is supplied. Heaters 1 and 2 thus operate at pressure P_2 and P_3 respectively. The amounts of steam m_1 and m_2 extracted from the turbine are such that at the exit from each of the heaters, the state is saturated liquid at the respective pressures.

$\therefore$ Turbine work, $W_T = 1(h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$

Pump work, $W_P = W_{P1} + W_{P2} + W_{P3}$

$= (1 - m_1 - m_2)(h_7 - h_6) + (1 - m_1)(h_9 - h_8) + 1(h_{11} - h_{10})$

$Q_H = (h_2 - h_{11})$; $\quad Q_L = (1 - m_1 - m_2)(h_5 - h_6)$

$\therefore$ Cycle efficiency, $\eta = \dfrac{Q_H - Q_L}{Q_H} = \dfrac{W_T - W_P}{Q_H}$

$$SSC = \frac{3600}{W_T - W_P} kg/kWh$$

In the Rankine cycle operating at the given pressure P_1 and P_4, the heat addition would have been from state 7 to state 2. By using two stages of regenerative feed water heating., feed water enters the boiler at state 11, instead of state 7, and heat addition is, therefore from state 11 to state 2. Therefore $(T_{m1})_{with\ regeneration} = \dfrac{h_2 - h_{11}}{S_2 - S_{11}}$

And $(T_{m1})_{without\ regeneration} = \dfrac{h_2 - h_7}{S_2 - S_7}$

Since $(T_{m1})_{\text{with regenerative}} > (T_{m1})_{\text{without regenerative}}$, the η of the regenerative cycle will be higher than that of the Rankine cycle.

The energy balance for heater 1,

$$m_1 h_3 + (1 - m_1) h_9 = 1\, h_{10}$$

$$\therefore m_1 = \frac{h_{10} - h_9}{h_3 - h_9} \qquad \text{--- (1)}$$

The energy balance for heater 2,

$m_2 h_4 + (1 - m_1 - m_2) h_7 = (1 - m_1) h_8$

Or $m_2 = (1 - m_1)\dfrac{(h_8 - h_7)}{(h_4 - h_7)}$ --- (2)

Above equations (1) and (2) can also be written alternatively as

$(1 - m_1)(h_{10} - h_9) = m_1 (h_3 - h_{10})$

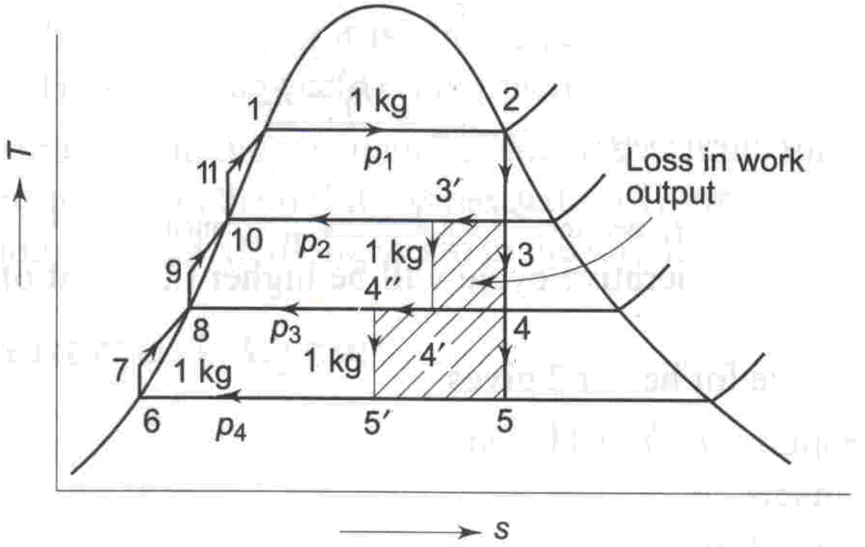
and $(1 - m_1 - m_2)(h_8 - h_7) = m_2 (h_4 - h_8)$

Energy gain of feed water = energy given off by vapour in condensation.

Heaters have been assumed to be adequately insulated and there is no heat gain from, or heat loss to, the surroundings.

In figure (a) path 2-3-4-5 represents the states of a decreasing mass of fluid.

For 1kg of steam, the states would be represented by the path $2\text{-}3^1\text{-}4^{11}\text{-}5^1$. [Figure (b)].



(b) Regenerative cycle on T-s plot for unit mass of fluid

We have $W_T = (h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$

$= (h_2 - h_3) + (h_3^1 - h_4^1) + (h_4^{11} - h_5^1)$ [From Figure b] --- (3)

The cycle $2 - 3 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11 - 2$ represents 1kg of working fluid. The heat released by steam condensing from 3 to 3^1 is utilized in heating up the water from 9 to 10.

$\therefore$ $1 (h_3 - h_3^1) = 1 (h_{10} - h_9)$ --- (4)

Similarly, $1 (h_4^1 - h_4^{11}) = 1 (h_8 - h_7)$ --- (5)

From equation (3), (4) and (5),

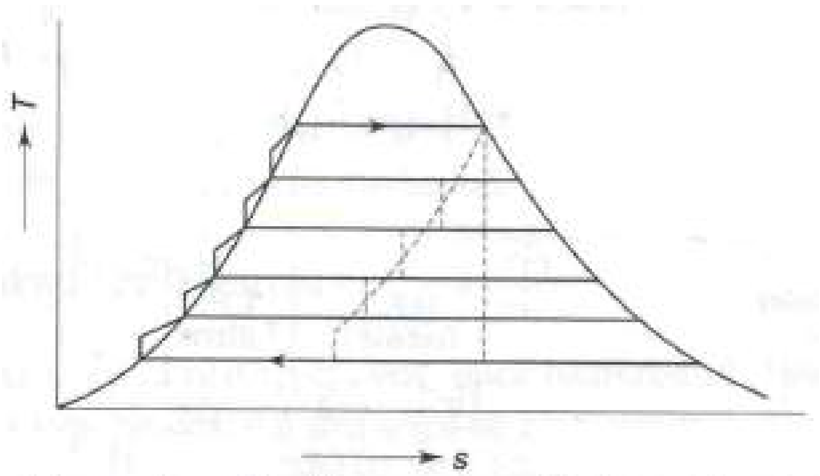
$W_T = (h_2 - h_5^1) - (h_3 - h_3^1) - (h_4^1 - h_4^{11})$

$$= (h_2 - h_5^1) - (h_{10} - h_9) - (h_8 - h_7) \qquad \text{--- (6)}$$

Also from Ideal regenerative cycle, [Previous article]

$$W_T = (h_2 - h_3^1) - (h_1^1 - h_1) \qquad \text{--- (1)}$$

The similarity of equations (6) and equation (1) from previous article is notices. It is seen that the stepped cycle $2 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11$ approximates the ideal regenerative cycle in Figure (1) [previous article] and that a greater no. of stages would give a closer approximation. Thus the heating of feed water by steam 'bled' from the turbine, known as regeneration, "Carnotizes" the Rankine cycle.



Regenerative cycle with many stages of feedwater heating

The heat rejected Q_L in the cycle decreases from $(h_5 - h_6)$ to $(h_5^1 - h_6)$. There is also loss in work output by the amount (area under $3 - 3^1$ + area under $4^1 - 4^{11}$ – area under $5 - 5^1$) as shown by the hatched area in Figure (b). So the steam rate increases by regeneration i.e., more steam has to circulate per hour to produce unit shaft output.

Reheat – Regenerative cycle:

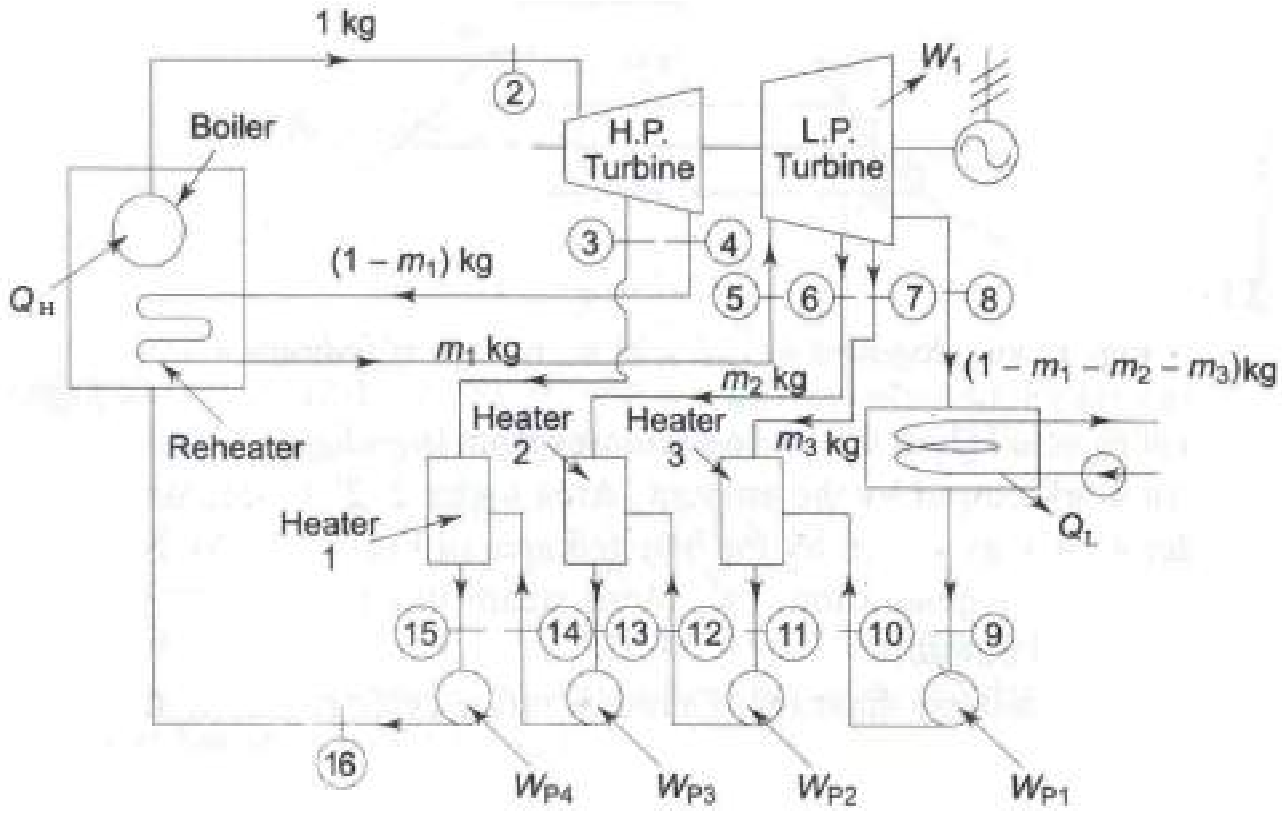


Fig. *Reheat-regenerative cycle flow diagram*

Reheat – regenerative cycle flow diagram (Three-stages of feed water heating)

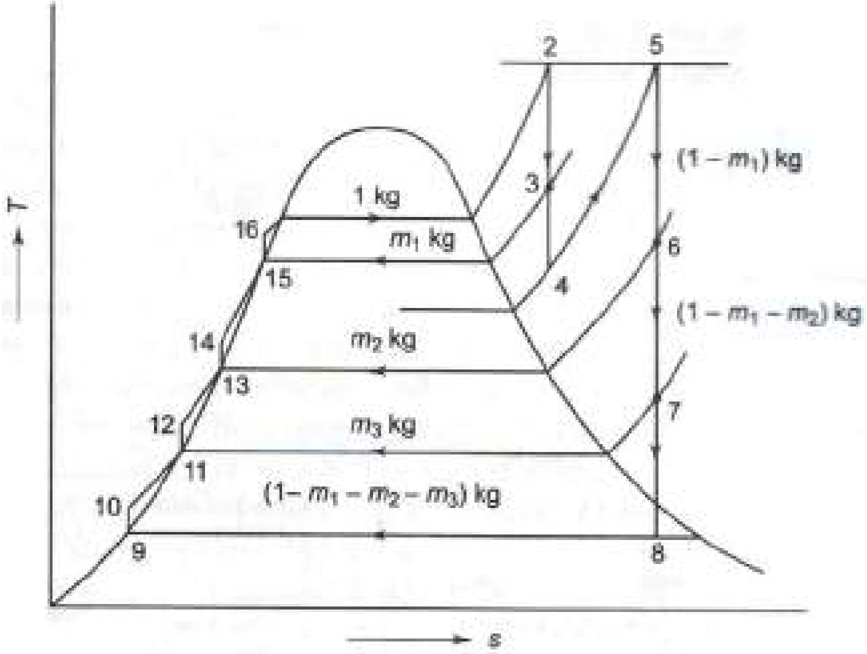


Fig. *T–s diagram of reheat-regenerative cycle*

The reheating of steam is employed when the vapourization pressure is high reheat alone on the thermal η is very small. ∴Regeneration or the heating up of feed water by steam extracted from the turbine will effect in more increasing in the η_{th}.

Turbine work, $W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1)(h_4 - h_5) + (1 - m_1 - m_2)(h_5 - h_6)$
$+ (1 - m_1 - m_2 - m_3)(h_6 - h_7)$ kJ/kg
Pump work, $W_P = (1 - m_1 - m_2 - m_3)(h_9 - h_8) + (1 - m_1 - m_2)(h_{11} - h_{10})$
$+ (1 - m_1)(h_{13} - h_{12}) + 1 (h_{15} - h_{14})$ kJ/kg

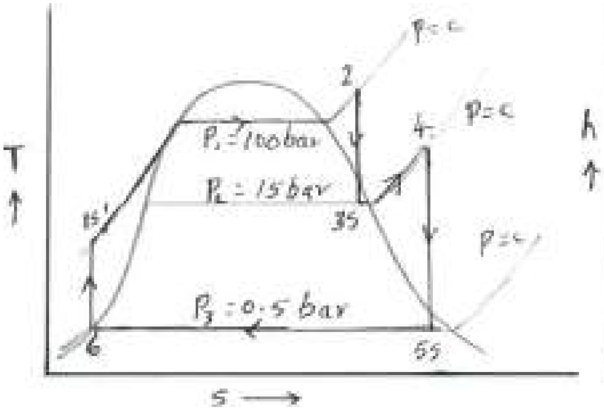
Heat added, $Q_H = (h_1 - h_{15}) + (1 - m_1)(h_4 - h_3)$ kJ/kg
Heat rejected, $Q_L = (1 - m_1 - m_2 - m_3)(h_7 - h_8)$ kJ/kg

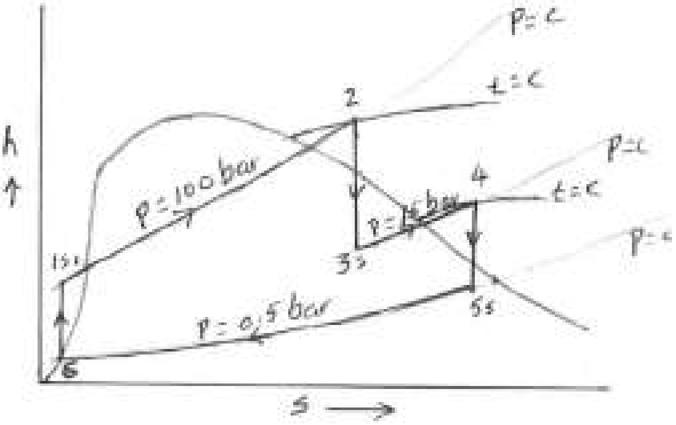
The energy balance of heaters 1, 2 and 3 gives
$m_1 h_2 + (1 - m_1) h_{13} = 1 \times h_{14}$
$m_2 h_5 + (1 - m_1 - m_2) h_{11} = (1 - m_1) h_{12}$
$m_3 h_6 + (1 - m_1 - m_2 - m_3) h_9 = (1 - m_1 - m_2) h_{10}$
From which m_1, m_2 and m_3 can be evaluated

Numerical Problems:

1. **An ideal reheat cycle utilizes steam as the working fluid. Steam at 100 bar, 400^{0}C is expanded in the HP turbine to 15 bar. After this, it is reheated to 350^{0}C at 15 bar and is then expanded in the LP turbine to the condenser pressure of 0.5 bar. Determine the thermal η and steam rate.**

Solution:





From steam tables
P = 100 bar t = 400^{0}C = v = 0.026408m^3/kg
h = 3099.9 kJ/kg
S = 6.2182 kJ/kg-K

P = 15 bar t_S = 192.28^{0}C, v_f = 0.0011538m^3/kg, v_g = 0.13167m^3/kg
h_f = 844.6 kJ/kg, h_{fg} = 1845.3 kJ/kg, h_g = 2789.9 kJ/kg
s_f = 2.3144kJ/kg-K, s_{fg} = 4.1262 kJ/kg-K, s_g = 6.4406 kJ/kg-K

P = 0.5 bar t_S = 81.35^{0}C, v_f = 0.0010301 m^3/kg, v_g = 3.2401 m^3/kg

$h_f = 340.6$ kJ/kg, $h_{fg} = 2305.4$ kJ/kg $h_g = 2646.0$ kJ/kg
$s_f = 1.0912$ kJ/kg-K $s_{fg} = 6.5035$ kJ/kg-K, $s_g = 7.5947$ kJ/kg-K

$h_2 = 3099.9$ kJ/kg,
Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$
$6.2182 = 2.3144 + x_{3S}\ (4.1262)$
$\therefore x_{3S} = 0.946$
$\therefore h_{3S} = 844.6 + x_{3S}\ (1845.3)$
$= 2590.44$ kJ/kg

$\therefore$ Expansion of steam in the HP turbine $= h_2 - h_{3S}$
$= 3099.9 - 2590.44$
$= 509.46$ kJ/kg

$P = 15$ bar, $t = 350^0C =$ $v = 0.18653$
$h = 3148.7$
$s = 7.1044$

Expansion of steam in the LP cylinder $= h_4 - h_{5s}$
$h_4 = 3148.7$ kJ/kg

To find h_{5s}:
We have $S_4 = S_{5s}$
$7.1044 = S_{f5} + x_{5S}\ S_{fg5}$
$= 1.0912 + x_{5S}\ (6.5035)$
$\therefore x_{5S} = 0.925$
$\therefore h_{5s} = 340.6 + 0.925\ (2305.4) = 2473.09$ kJ/kg
$\therefore$ Expansion of steam in the LP turbine $= 3148.7 - 2473.09$
$= 675.61$ kJ/kg

$h_6 = h_f$ for $P_3 = 0.5$ bar i.e., $h_6 = 340.6$ kJ/kg
Pump work, $W_P = h_{1s} - h_6$ $= v_{f5}\ (P_3 - P_1) = 0.0010301\ (100 - 0.501 \times 10^5)$
$= 10.249$ kJ/kg
$\therefore h_{1s} = 350.85$ kJ/kg

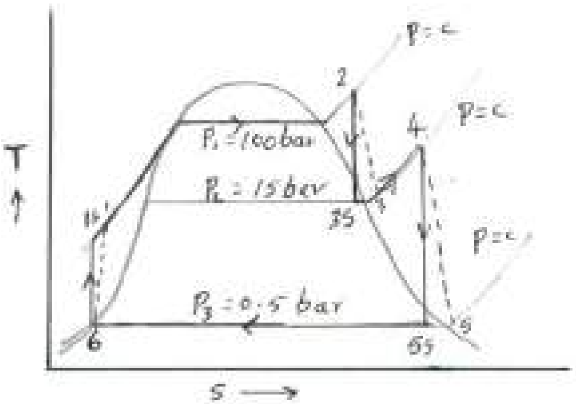
$\therefore$ Heat supplied, Q_H $= (h_2 - h_{1S}) + (h_4 - h_{3S})$
$= (3099.9 - 350.85) + (3148.7 - 2590.44)$
$= 2749.05$ kJ/kg $+ 558.26$
$= 3307.31$ kJ/kg

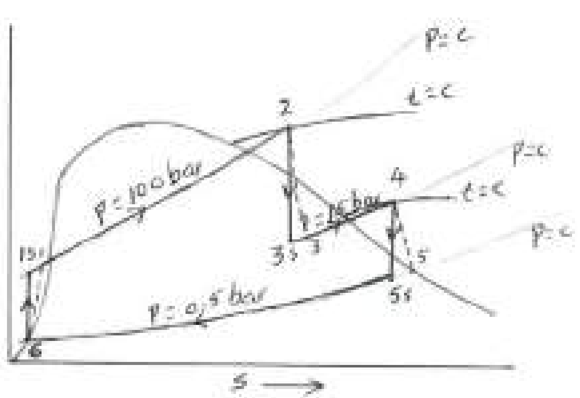
$$\therefore \quad \eta_{th} = \frac{W_{net}}{Q_H} = \frac{(W)_{HP} + (W)_{LP} - W_P}{Q_H}$$

$$= \frac{509.46 + 675.61 - 10.25}{3307.31} = 0.355$$

Steam rate, SSC $= \dfrac{3600}{W_{net}} = 3.064 kg/kWh$

1. b) When η of the HP turbine, LP turbine and feed pump are 80%, 85% and 90% respectively.





$$\eta_{tHP} = \frac{h_2 - h_3}{h_2 - h_{3s}} = 0.8 = \frac{3099.9 - h_3}{3099.9 - 2590.44}$$

$\therefore$ h$_3$ = 2692.33 kJ/kg

$$\eta_{tLP} = \frac{h_4 - h_5}{h_4 - h_{5s}} = 0.85 = \frac{3148.7 - h_5}{3148.7 - 2473.09}$$

$\therefore$ h$_5$ = 2574.43 kJ/kg

$$\eta_P = \frac{h_{15} - h_6}{h_1 - h_6} = 0.9 = \frac{350.85 - 340.6}{h_1 - 340.6}$$

$\therefore$ h$_1$ = 351.99 kJ/kg

$$\therefore \eta_{th} = \frac{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)}{(h_2 - h_1) + (h_4 - h_3)}$$

$$= \frac{(3099.9 - 2692.33) + (3148.7 - 2574.43) - (351.99 - 340.6)}{(3099.9 - 351.99) + (3148.7 - 2692.33)}$$

= 0.303 or 30.3%

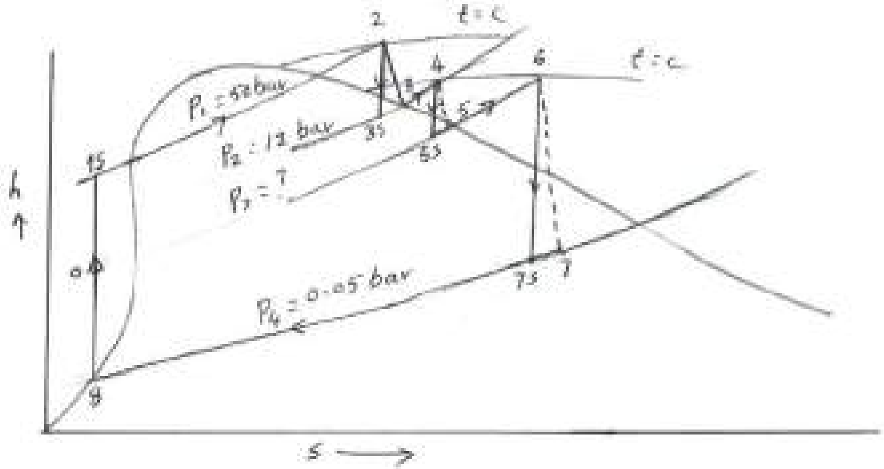
$\therefore$ SSC = 3.71 kg/kWh

Using Mollier-chart: h_2 = 3095 kJ/kg, h_{3s} = 2680 kJ/kg, h_4 = 3145 kJ/kg
h_5 = 2475 kJ/kg, h_6 = 340.6 kJ/kg (from steam tables)
W_P = 10.249 kJ/kg

2. **Steam at 50 bar, 350^{0}C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280^{0}C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^{0}C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit**

of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



$P_1 = 50$ bar $t_2 = 350^0$C $P_2 = 12$ bar $t_4 = 280^0$C, $t_6 = 280^0$C

$P_3 = ?$ $P_4 = 0.05$ bar

From Mollier diagram

$h_2 = 3070$kJ/kg $h_{3s} = 2755$ kJ/kg $h_3 = 2780$ kJ/kg $h_4 = 3008$ kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$

$= 0.921$

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$h_2 - h_3 = h_4 - h_5$

i.e., $3070 - 2780 = 3008 - h_5$

$\therefore h_5 = 2718$ kJ/kg

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar,

Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93\text{kJ/kg}$$

$\therefore$ At a pressure 0.05 bar, $h_7 = h_{f7} + x_7 h_{fg7}$

$2420.93 = 137.8 + x_7 (2423.8)$

$\therefore x_7 = 0.941$

Total power output $= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7)$

$= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93)$

$= 1197.07$ kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

For $P_4 = 0.05$ bar from steam tables, $h_8 = 137.8$ kJ/kg;

$W_P = 0.0010052 (50 - 0.05) 10^2 = 5.021$ kJ/kg

$= h_8 - h_{1s}$

$$\therefore h_{1s} = 142.82 \text{ kJ/kg}$$

$$\begin{aligned}\text{Heat supplied, } Q_H &= (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5)\\ &= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718)\\ &= 3475.18 \text{ kJ/kg}\end{aligned}$$

$W_{net} = W_T - W_P = 1197.07 - 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$$\therefore T_m = 465.6 \text{ K}$$

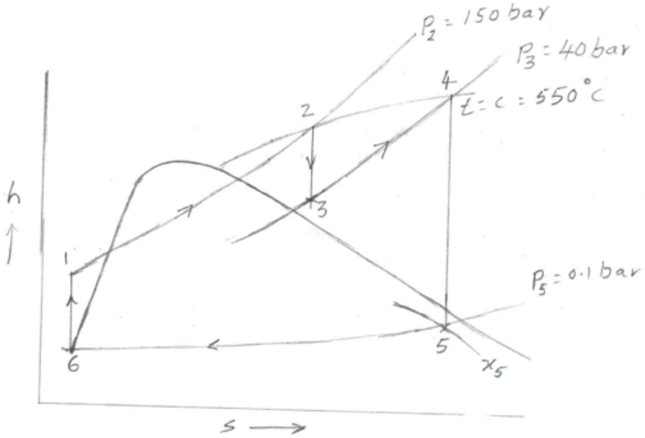
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02 kg/kWh$$

3. A steam power station uses the following cycle: Steam at boiler outlet – 150 bar; reheat at 40 bar, 550^0C; condenser at 0.1 bar. Using Mollier chart and assuming that all processes are ideal, find (i) quality at turbine exhaust (ii) cycle η (iii) steam rate.

Solution:



P_2 = 150 bar $t_2 = 550^0$C P_3 = 40 bar $t_3 = 550^0$C

P_5 = 0.1 bar

From Mollier diagram   i.e., h-s diagram

$$h_2 = h\Big|_{150\,bar,550^0C} = 3450 kJ/kg$$

$$h_4 = h\Big|_{40\,bar,550^0C} = 3562 kJ/kg$$

h_3 = 3050 kJ/kg

h_5 = 2290 kJ/kg

x_5 = 0.876 kJ/kg

h_6 can not determined from h-s diagram, hence steam tables are used.

$$h_6 = h_f\Big|_{0.1bar} = 191.8 kJ/kg$$

Process 6-1 is isentropic pump work i.e., $W_{P1} = v\int dP$

$= 0.0010102\ (40 - 01)\ 10^5/10^3 = 4.031$ kJ/kg

$= (h_1 - h_6)$

$\therefore h_1 = 195.8$ kJ/kg

(i) Quality of steam at turbine exhaust = $x_5 = 0.876$

$$(ii)\ \eta_{cycle} = \frac{W_T - W_P}{Q_H}$$

Turbine work, $W_T = W_{T1} + W_{T2}$

$= (h_2 - h_3) + (h_4 - h_5)$

$= (3450 - 3050) + (3562 - 2290)$

$= 1672$ kJ/kg

$Q_H = Q_1 + Q_R = (h_2 - h_1) + (h_4 - h_3)$

$= (3450 - 195.8) + (3562 - 3050)$

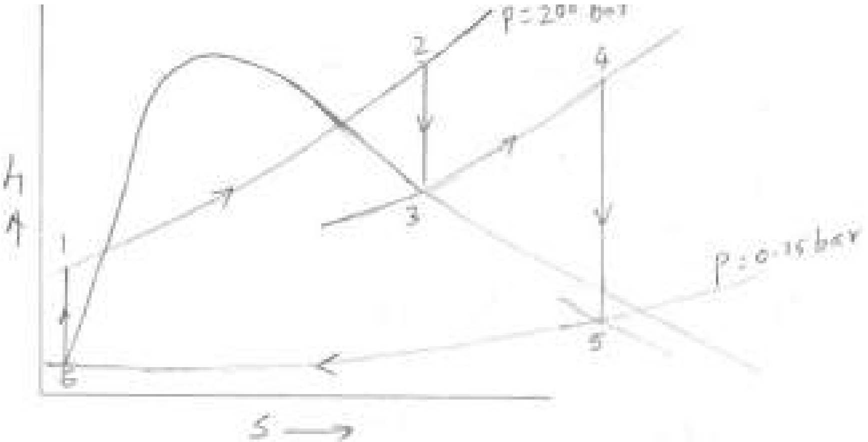
$= 3766.2$ kJ/kg

$$\therefore \eta_{cycle} = \frac{1672 - 4.031}{3766.2} = \frac{1667.97}{3766.2} = 0.443$$

$$(iii) \quad Steam\ rate = \frac{3600}{1667.97} = 2.16 kg / kWh$$

4. An ideal Rankine cycle with reheat is designed to operate according to the following specification. Pressure of steam at high pressure turbine = 20 MPa, Temperature of steam at high pressure turbine inlet = 550^{0}C, Temperature of steam at the end of reheat = 550^{0}C, Pressure of steam at the turbine exhaust = 15 KPa. Quality of steam at turbine exhaust = 90%. Determine (i) the pressure of steam in the reheater (ii) ratio of pump work to turbine work, (iii) ratio of heat rejection to heat addition, (iv) cycle η.

Solution:



P_2 = 200 bar t_2 = 550^{0}C t_4 = 550^{0}C P_5 = 0.15 bar x_5 = 0.9

From Mollier diagram,

h_2 = 3370 kJ/kg

h_3 = 2800 kJ/kg

h_4 = 3580 kJ/kg

h_5 = 2410 kJ/kg

x_5 = 0.915

$P_3 = P_4$ = 28 bar

But given in the data x_5 = 0.9

From steam tables h_6 = 226 kJ/kg

Pump work $W_P = v\int dP$

$$= 0.001014\ (200 - 0.15)\ 10^5/10^3$$

$= 20.26$ kJ/kg

But $W_P = h_1 - h_6$ $\quad \therefore h_1 = 246.26$ kJ/kg

(i) Pressure of steam in the reheater = 28 bar

(ii) Turbine work $W_T = (h_2 - h_3) + (h_4 - h_5)$

$= (3370 - 2800) + (3580 - 2410)$

$= 1740$ kJ/kg

$\therefore$ Ratio of $\dfrac{W_P}{W_T} = 0.0116 \quad i.e., \quad 1.2\%$

(iii) $Q_L = (h_5 - h_6) = (2410 - 226) = 2184$ kJ/kg

$Q_H = (h_2 - h_1) + (h_4 - h_3)$

$= (3370 - 226) + (3580 - 2800)$

$= 3924$ kJ/kg

$\therefore \dfrac{Q_L}{Q_H} = 0.5565 \quad i.e., \quad 55.65\%$

$$(iv)\ \eta_{cycle} = \frac{W_{net}}{Q_{Total}} = \frac{(1740 - 20.26)}{3924} = 0.4383 \quad i.e., \quad 43.8\%$$

Feedwater Heaters (FWH)

A practical Regeneration process in steam power plants is accomplished by extracting or bleeding, steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feed water instead. The device where the feedwater heated by regeneration is called a Regenerator or a Feedwater Heater (FWH).

A feedwater heater is basically a heat exchanger where heat is transferred from the steam to the feedwater either by mixing the two streams (open feedwater heaters) or without mixing them (closed feedwater heaters).

Open Feedwater Heaters

An open (or direct-contact) feedwater heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.

The advantages of open heater are simplicity, lower cost, and high heat transfer capacity. The disadvantage is the necessity of a pump at each heater to handle the large feedwater stream.

Closed Feedwater Heaters

In closed feedwater heater, the heat is transferred from the extracted steam to the feedwater without mixing taking place. The feedwater flows through the tubes in the heater and extracted steam condenses on the outside of the tubes in the shell. The heat released from the condensation is transferred to the feedwater through the walls of the tubes. The condensate (saturated water at the steam extraction pressure), some times called the heater-drip, then passes through a trap into the next lower pressure heater. This, to some extent, reduces the steam required by that heater. The trap passes only liquid and no vapour. The drip from the lowest pressure heater could similarly be trapped to the condenser, but this would be throwing away energy to the condenser cooling water. The avoid this waste, the drip pump feed the drip directly into the feedwater stream.

A closed heaters system requires only a single pump for the main feedwater stream regardless of the number of heaters. The drip pump, if used is relatively small. Closed heaters are costly and may not give as high a feedwater temperature as do open heaters.

In most steam power plants, closed heaters are favoured, but atleast one open heater is used, primarily for the purpose of feedwater deaeration. The open heater in such a system is called deaerator.

Note: The higher the number of heater used, the higher will be the cycle efficiency. The number of heater is fixed up by the energy balance of the whole plant when it is found that the cost of adding another does not justify the saving in Q_H or the marginal increase in cycle efficiency. An increase in feedwater temperature may, in some cases, cause a reduction in boiler efficiency. So the number of heaters get optimized. Five feedwater heaters are often used in practice.

Characteristics of an Ideal working fluid

The maximum temperature that can be used in steam cycles consistent with the best available material is about 600^0C, while the critical temperature of steam is 375^0C, which necessitates large superheating and permits the addition of only an infinitesimal amount of heat at the highest temperature.

The desirable characteristics of the working fluid in a vapour power cycle to obtain best thermal η are as follows:

a) The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of evaporation at that pressure.
b) The saturation pressure at the temperature of heat rejection should be above atmosphere pressure so as to avoid the necessity of maintaining vacuum in the condenser.
c) The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to the boiling point.
d) The saturation vapour line of the T-S diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.
e) The freezing point of the fluid should be below room temperature, so that it does not get solidified while flowing through the pipe lines.
f) The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.
g) The fluid should be nontoxic, non corrosive, not excessively viscous, and low in cost.

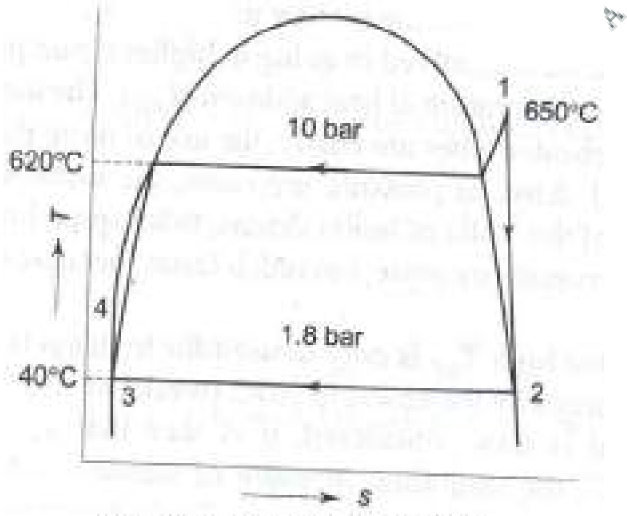
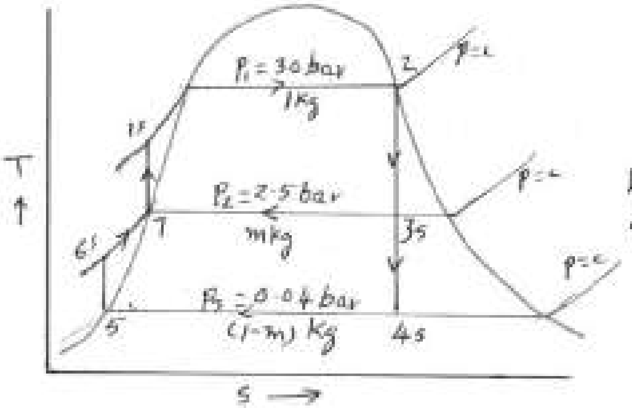


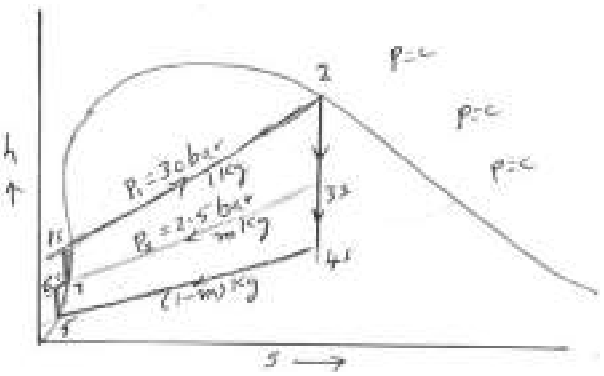
Fig. T-S diagram for an Ideal Working Fluid for a Vapour Power Cycle

Numerical Problems:

1. **An ideal regenerative cycle operates with dry saturated steam, the maximum and minimum pressures being 30 bar and 0.04 bar respectively. The plant is installed with a single mixing type feed water heater. The bled steam pressure is 2.5 bar. Determine (a) the mass of the bled steam, (b) the thermal η of the cycle, and (c) SSC in kg/kWh.**

Solution:





$P_1 = 30$ bar $\quad P_2 = 2.5$ bar $\quad P_3 = 0.04$ bar

From steam tables, For $P_1 = 30$ bar, $\quad h_2 = 2802.3$ kJ/kg, $\quad S_2 = 6.1838$ kJ/kg-^{0}k

But $S_2 = S_{3s}$ i.e., $6.1838 = 1.6072 + x_3 (5.4448)$

$\therefore x_3 = 0.841$

$\therefore h_3 = 535.4 + 0.841 (2281.0)$

$= 2452.68$ kJ/kg

Also $S_2 = S_{4s}$ i.e., $6.1838 = 0.4225 + x_4 (8.053)$

$\therefore x_4 = 0.715$

$\therefore h_4 = 121.4 + 0.715 (2433.1)$

$= 1862.1$ kJ/kg

At $P_3 = 0.04$ bar, $\quad h_5 = 121.4$ kJ/kg, $\quad v_5 = 0.001004$ m^3/kg

$\therefore$Condensate pump work $= (h_6 - h_5) = v_5 (P_2 - P_3)$

$= 0.001004 (2.5 - 0.04) (10^5/10^3)$

$= 0.247$ kJ/kg

$\therefore h_6 = 0.247 + 121.4 = 121.65$ kJ/kg

Similarly, $h_1 = h_7 + v_7 (P_1 - P_2) (10^5/10^3)$

$= 535.4 + 0.0010676 (30 - 2.5) 10^2$

$= 538.34$ kJ/kg

a) **Mass of the bled steam:**

Applying the energy balance to the feed water heater

$mh_3 + (1 - m)\, h_6 = 1\, (h_7)$

$$\therefore m = \frac{(h_7 - h_6)}{(h_3 - h_6)} = \frac{(535.4 - 121.65)}{(2452.68 - 121.65)} = 0.177 kg/kg \text{ of steam}$$

b) **Thermal η:**

Turbine work, W_T $= 1\, (h_2 - h_{3s}) + (1 - m)\, (h_3 - h_{4s})$
$= 1\, (2802.3 - 2452.65) + (1 - 0.177)\, (2452.68 - 1862.1)$
$= 835.67$ kJ/kg

Pump work, W_P $= (1 - m)\, (h_{6s} - h_5) + 1\, (h_{1s} - h_7)$
$= (1 - 0.177)\, (121.65 - 121.4) + 1\, (538.34 - 535.4)$
$= 3.146$ kJ/kg

$\therefore W_{net} = W_T - W_P = 832.52$ kJ/kg

Heat supplied, Q_H $= 1\, (h_2 - h_{1s})$
$= 1\, (2802.3 - 538.34)$
$= 2263.96$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{832.52}{2263.96} = 0.368 \text{ or } \mathbf{36.8\%}$$

c) **SSC:**

$$SSC = \frac{3600}{W_{net}} = 4.324 kg/kWh$$

2. **In problem (3), also calculate the increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration)**

Solution: Tm_1 (with regeneration) $= \dfrac{h_2 - h_1}{S_2 - S_1} = \dfrac{2263.96}{(6.1838 - 1.6072)} = 494.68k$

Tm_1 (without regeneration) $= \dfrac{h_2 - h_6}{S_2 - S_6} = \dfrac{2802.3 - 121.65}{(6.1838 - 0.4225)} = 465.29k$

$\therefore$ Increase in Tm_1 due to regeneration = 494.68 – 465.29 = **29.39^0K**

W_T (without regeneration) = $h_2 - h_4$ = 2802.3 – 1862.1 = 940.2 kJ/kg

W_P (without regeneration) = $(h_1 - h_5) = v_5\, (30 - 0.04)\, 10^2$
$= 0.001004\, (29.96)\, 10^2 = 3.01$ kJ/kg

$\therefore h_1 = 3.01 + 121.4 = 124.41$ kJ/kg

$$\therefore \eta_{th} \text{ (without regeneration) } = \frac{W_{net}}{Q_H} = \frac{(940.2-3.01)}{2802.3-124.41} = 0.349$$

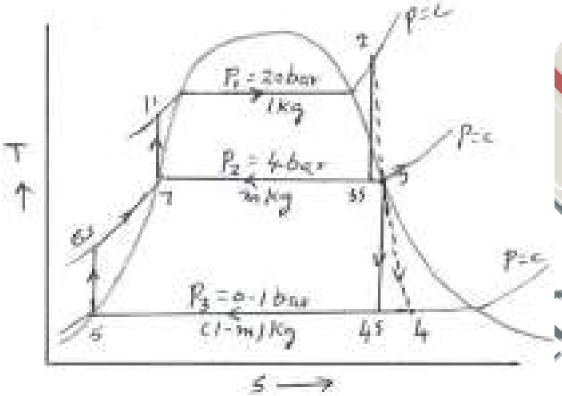
$\therefore$ Increase in η_{th} due to regeneration = 0.368 – 0.349 = 0.018 i.e., 1.8%

Steam rate (without regeneration) = 3.84 kg/kWh

$\therefore$ Increase in steam rate due to regeneration = 4.324 – 3.84
= 0.484 kg/kWh

3. **Steam at 20 bar and 300^{0}C is supplied to a turbine in a cycle and is bled at 4 bar. The bled-steam just comes out saturated. This steam heats water in an open heater to its saturation state. The rest of the steam in the turbine expands to a condenser pressure of 0.1 bar. Assuming the turbine efficiency to be the same before and after bleeding, find: a) the turbine η and the steam quality at the exit of the last stage; b) the mass flow rate of bled steam 1kg of steam flow at the turbine inlet; c) power output / (kg/s) of steam flow; and d) overall cycle η.**

Solution:



P_1 = 20 bar t_1 = 300^{0}C P_2 = 4 bar P_3 = 0.1 bar

From steam tables,

For P_1 = 20 bar and t_1 = 300^{0}C

$v_2 = 0.12550$ $h_2 = 3025.0$ $S_2 = 6.7696$

For P_2 = 4 bar, $h_3 = 2737.6$, $t_s = 143.63$

$h_f = 604.7$, $h_{fg} = 2132.9$, $S_f = 1.7764$, $S_{fg} = 5.1179$, $S_g = 6.8943$

For P_2 = 0.1 bar, 45.83, 191.8, 2392.9, 2584.8, 0.6493, 7.5018, 8.1511

We have, $S_2 = S_{3s}$ i.e., 6.7696 = 1.7764 + x_3 (5.1179)

$\therefore x_3 = 0.976$

$\therefore h_{3s}$ = 604.7 + 0.976 (2132.9) = 2685.63 kJ/kg

$$\therefore \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3025 - 2737.6}{3025 - 2685.63} = 0.847$$

$S_3 = S_{4s}$ i.e., 6.8943 = 0.6493 + x_4 (7.5018)

$\therefore x_{4s} = 0.832$

$\therefore h_{4s}$ = 191.8 + 0.832 (2392.9) = 2183.81kJ/kg

But η_t is same before and after bleeding i.e., $\eta_t = \dfrac{h_3 - h_4}{h_3 - h_{4s}}$

$$\text{i.e., } 0.847 = \frac{2737.6 - h_4}{2737.6 - 2183.81}$$

$\therefore h_4 = 2268.54$ kJ/kg

$\therefore h_4 = h_{f4} + x_4 h_{fg4}$ $\quad \therefore x_4 = 0.868$

b) Applying energy balance to open heater, $mh_3 + (1 - m) h_{6s} = 1 (h_7)$

$$\therefore m = \frac{h_7 - h_6}{h_3 - h_6}$$

Condensate pump work, $(h_{6s} - h_5) = v_5 (P_3 - P_2)$

$= 0.0010102 (3.9) 10^2 = 0.394$ kJ/kg

$\therefore h_{6s} = 191.8 + 0.394 = 192.19$ kJ/kg

Similarly, $h_{1s} = h_7 + v_7 (P_1 - P_2)$

$= 604.7 + -.0010839 (16) 10^2 = 606.43$ kJ/kg

$$\therefore m = \frac{604.7 - 192.19}{2737.6 - 192.19} = 0.162$$

c) Power output or $W_T = (h_2 - h_3) + (1 - m) (h_3 - h_4)$

$= (3025 - 2737.6) + (1 - 0.162) (2737.6 - 2268.54)$

$= 680.44$ kJ/kg

For 1kg/s of steam, $\quad W_T = 680.44$ kW

d) Overall thermal efficiency, $\eta_0 = \dfrac{W_{net}}{Q_H}$

$W_P = (1 - m) (h_{6s} - h_5) + 1 (h_{1s} - h_7)$

$= (1 - 0162) (192.19 - 191.8) + 1 (606.43 - 604.7)$

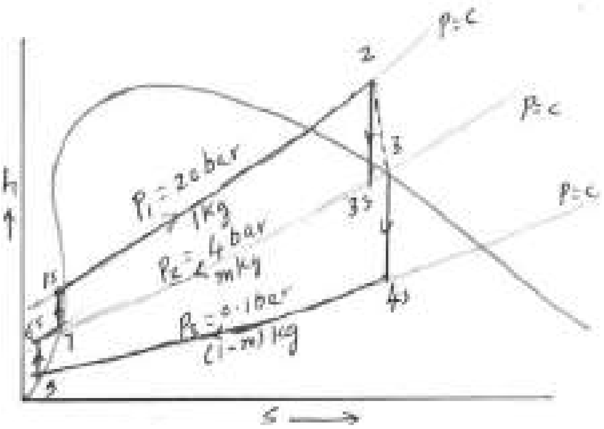
$= 2.057$ kJ/kg

$W_{net} = 680.44 - 2.057 = 678.38$ kJ/kg

$Q_H = 1 (h_2 - h_{1s}) = (3025 - 606.43) = 2418.57$ kJ/kg

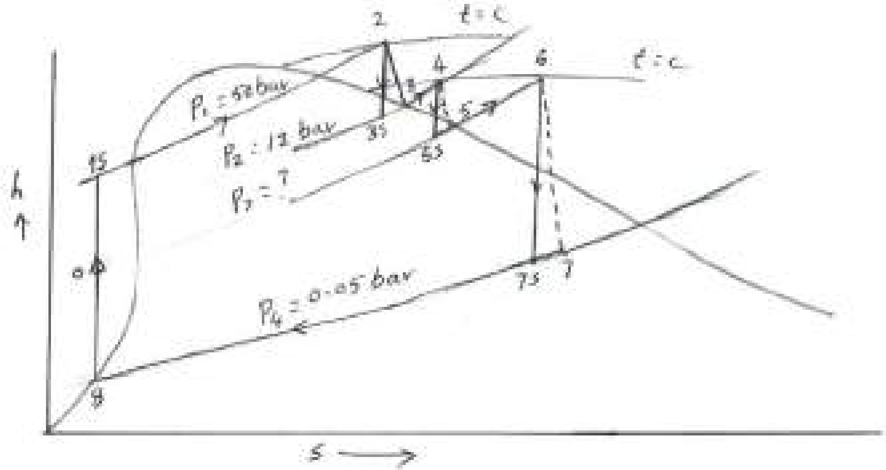
$$\therefore \eta_0 = \frac{678.38}{2418.57} = 0.2805$$

Using Moiller Diagram



4. **Steam at 50 bar, 350^{0}C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280^{0}C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^{0}C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.**

Solution:



$P_1 = 50$ bar $t_2 = 350^0$C $P_2 = 12$ bar $t_4 = 280^0$C, $t_6 = 280^0$C

$P_3 = ?$ $P_4 = 0.05$ bar

From Mollier diagram

$h_2 = 3070$kJ/kg $h_{3s} = 2755$ kJ/kg $h_3 = 2780$ kJ/kg $h_4 = 3008$ kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$

$= 0.921$

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$h_2 – h_3 = h_4 – h_5$

i.e., $3070 – 2780 = 3008 – h_5$

$\therefore h_5 = 2718$ kJ/kg

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar,
Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93\text{kJ/kg}$$

$\therefore$ At a pressure 0.05 bar, $h_7 = h_{f7} + x_7\, h_{fg7}$

$2420.93 = 137.8 + x_7\,(2423.8)$

$\therefore x_7 = 0.941$

Total power output $= (h_2 – h_3) + (h_4 – h_5) + (h_6 – h_7)$
$= (3070 – 2780) + (3008 – 2718) + (3038 – 2420.93)$
$= 1197.07$ kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

For $P_4 = 0.05$ bar from steam tables, $h_8 = 137.8$ kJ/kg;

$W_P = 0.0010052\,(50 – 0.05)\,10^2 = 5.021$ kJ/kg
$= h_8 – h_{1s}$

$\therefore h_{1s} = 142.82$ kJ/kg

Heat supplied, $Q_H = (h_2 – h_{1s}) + (h_4 – h_3) + (h_6 – h_5)$
$= (3070 – 142.82) + (3008 – 2780) + (3038 – 2718)$
$= 3475.18$ kJ/kg

$W_{net} = W_T – W_P = 1197.07 – 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$\therefore T_m = 465.6$ K

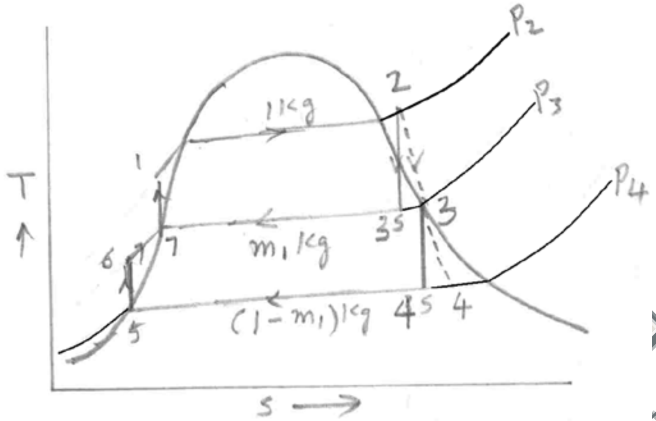
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02kg / kWh$$

5. **Steam at 30 bar and 350^{0}C is supplied to a steam turbine in a practical regenerative cycle and the steam is bled at 4 bar. The bled steam comes out as dry saturated steam and heats the feed water in an open feed water heater to its saturated liquid state. The rest of the steam in the turbine expands to condenser pressure of 0.1 bar. Assuming the turbine η to be same before and after bleeding determine (i) the turbine η, (ii) steam quality at inlet to condenser, (iii) mass flow rate of bled steam per unit mass rate at turbine inlet and (iv) the cycle η.**

Solution:



P_2 = 30 bar $t_2 = 350^0$C P_3 = 4 bar P_4 = 0.1 bar

$h_3 = h_g$ at P_3 = 4 bar, = 2737.6 kJ/kg

From superheated steam tables,

$$h_2 = h_3 = h_g\Big|_{P_3=4bar} = 2737.6\ kJ/kg$$

$$h_2 = h\Big|_{P_2=30bar\ \&\ t_2=350^0C} = 3117.5\ kJ/kg \text{ and } S_2 = 6.7471 \text{ kJ/kg-K}$$

$$h_5 = h_f\Big|_{P_5=0.1bar} = 191.8\ kJ/kg$$

$$h_7 = h_f\Big|_{P_7=4bar} = 604.7\ kJ/kg$$

Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$

$$6.7471 = 1.7764 + x_{3S}\,(5.1179)$$

$$\therefore x_{3S} = 0.971$$

$\therefore h_{3S} = h_{f3} + x_{3S}\, h_{fg3}$

$= 604.7 + 0.971\,(2132.9)$

$= 2676.25$ kJ/kg

Process 3-4s is isentropic i.e., $S_3 = S_{4S}$

i.e., $6.8943 = 0.6493 + x_{4S}\,(7.5018)$

$\therefore x_{4S} = 0.832$

$\therefore h_{4S} = 191.8 + 0.832\,(2392.9) = 2183.8$ kJ/kg

Given, η_t (before bleeding) = η_t (after bleeding)

We have, η_t (before bleeding) $= \dfrac{h_2 - h_3}{h_2 - h_{3S}} = \dfrac{3117.5 - 2737.6}{3117.5 - 2676.25} = 0.86$

$$\therefore 0.86 = \frac{h_3 - h_4}{h_3 - h_{4S}} = \frac{2737.6 - h_4}{2737.6 - 2183.8} \quad \therefore h_4 = 2261.33\,kJ/kg$$

But $h_4 = h_{f4} + x_4\, h_{fg4}$

$2261.33 = 191.8 + x_4\,(2392.9)$

$\therefore x_4 = 0.865$

i.e., Dryness fraction at entry to condenser = x_4 = **0.865**

iii) Let m kg of steam is bled. Applying energy balance to FWH,

$m h_3 + (1 - m)\, h_6 = h_7$

We have $W_{P1} = (h_6 - h_5) = v\int dP$

$= 0.0010102\,(4 - 0.1)\,10^5/10^3$

$= 0.394$ kJ/kg

$\therefore h_6 = 0.394 + 191.8 = 192.19$ kJ/kg

Substituting,

$m\,(2737.6) + (1 - m)\,192.19 = 604.7$

$\therefore m = 0.162$ kg

Also, $W_{P2} = (h_1 - h_7) = v\int dP$

$= 0.0010839 \times (30 - 4)\,10^2$

$= 2.82$ kJ/kg

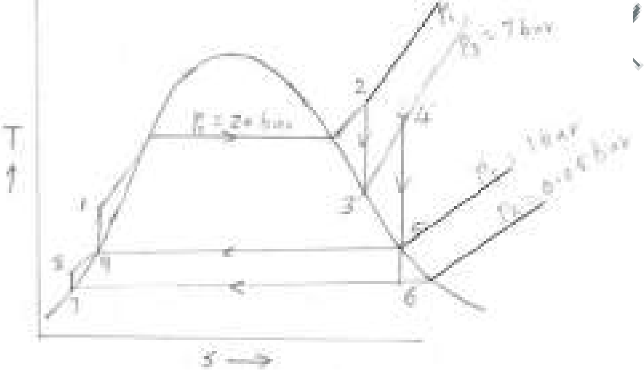
$\therefore h_1 = 2.82 + 604.7 = 607.52$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_T = W_P}{Q_H} = \frac{\left[(h_2 - h_3) + (1 - m)(h_3 - h_4)\right] - \left[(1 - m)(h_6 - h_5) + (h_1 - h_2)\right]}{(h_2 - h_1)}$$

$\eta_{cycle} = 0.31$

6. **In an ideal reheat regenerative cycle, the high pressure turbine receives steam at 20 bar, 300^{0}C. After expansion to 7 bar, the steam is reheated to 300^{0}C and expands in an intermediate pressure turbine to 1 bar. A fraction of steam is now extracted for feed water heating in an open type FWH. The remaining steam expands in a low pressure turbine to a final pressure of 0.05 bar. Determine (i) cycle thermal η, (ii) specific steam consumption, (iii) quality of steam entering condenser.**

Solution:



$$h_2 = h\Big|_{20\,bar,\,300^0 C} = 3025\,kJ/kg \text{ and } s_2 = 6.7696 \text{ kJ/kg-K}$$

Process 2-3 is isentropic

i.e., $S_2 = S_3$

$6.7696 = 1.9918 + x_3\,(4.7134)$

$\therefore x_3 = 1.014$

i.e., state 3 can be approximated as dry saturated.

$\therefore h_3 = h\Big|_{7\,bar,\,dry\,sat.} = 2762$ kJ/kg

$\therefore h_4 = h\Big|_{7\,bar,\,300^0C} = 3059.8 kJ/kg$ and $s_4 = 7.2997$ kJ/kg-K

Process 4-5 is isentropic i.e., $S_4 = S_5$

$$7.2997 = 1.3027 + x_5\,(6.0571)$$

$$\therefore x_5 = 0.99$$

$\therefore h_5 = h_{f5} + x_5\,h_{fg5} = 417.5 + 0.99\,(2257.9) = 2652.9$ kJ/kg

Process 5-6 is isentropic i.e., $S_5 = S_6$

$$7.2997 = 0.4763 + x_6\,(7.9197)$$

$$\therefore x_6 = 0.862$$

$\therefore h_6 = 137.8 + 0.862\,(2423.8) = 2226.1$ kJ/kg

$h_7 = h_7 = h_f\Big|_{0.05\,bar} = 137.8$ kJ/kg

Neglecting W_{P1}, $h_8 = h_7$, Also neglecting W_{P2}, $h_9 = h_1$

$\therefore h_9 = h_f\Big|_{1\,bar} = 417.5$ kJ/kg

Applying energy balance to FWH

$$mh_5 + (1 - m)\,h_8 = h_9$$

i.e., $m\,(2652.9) + (1 - m)\,137.8 = 417.5$ $\therefore m = 0.111$ kg/kg of steam

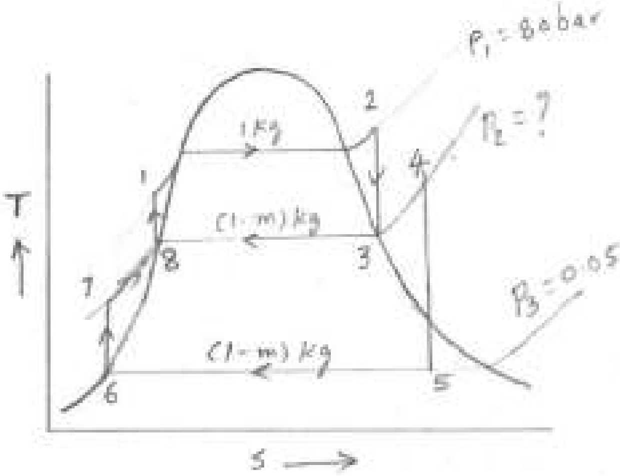
$$(i)\quad \eta_C = \frac{(h_2 - h_3) + (h_4 - h_5) + (1 - m)(h_5 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} = 0.35$$

$$(ii)\quad SSC = \frac{3600}{W_{net}} = 3.57 kg / kWh$$

(iii) Quality of steam entering condenser, $x_6 = 0.862$

7. **The net power output of a regenerative – reheat cycle power plant is 80mW. Steam enters the high pressure turbine at 80 bar, 500^0C and expands to a pressure P_2 and emerges as dry vapour. Some of the steam goes to an open feed water heater and the balance is reheated at 400^0C at constant pressure P_2 and then expanded in the low pressure turbine to 0.05 bar. Determine (i) the reheat pressure P_2, (ii) the mass of bled steam per kg boiler steam, (iii) the steam flow rate in HP turbine, (iv) cycle η. Neglect pump work. Sketch the relevant lines on h-s diagram. Assume expansion in the turbines as isentropic.**

Solution:



$P = 80000$ kW $P_1 = 80$ bar $t_2 = 500^0$C $P_2 = ?$ $t_3 = 400^0$C

$P_3 = 0.05$ bar $m = ?$ $\dot{m}_S = ?$ $\eta_{cycle} = ?$

$$h_2 = h\Big|_{80\,bar,\,500^0C} = 3398.8\,kJ/kg \text{ and } s_2 = 6.7262$$

Process 2-3 is isentropic i.e., $S_2 = S_3 = 6.7262$ kJ/kg-K

Given state 3 is dry saturated i.e., $S_3 = 6.7262 = S_g\Big|_{P_2}$

From table A – 1, for dry saturated steam, at P = 6.0 bar, $S_g = 6.7575$

and at P = 7.0 bar, $S_g = 6.7052$

Using linear interpolation,

$$\Delta P = \frac{6.0 - 7.0}{(6.7575 - 6.7052)} x(6.7262 - 6.7052) = 0.402\ bar$$

$\therefore$ (i) $P_2 = 6 + 0.402 = 6.402$ bar

$$\therefore h_3 = h\Big|_{P_2 = 6.4 bar}$$

From table A – 1, For P = 6 bar $h_g = 2755.5$ $S_g = 6.7575$

For P = 7 bar, $h_g = 2762.0$ $S_g = 6.7052$

$$\therefore For\ P = 6.4\ bar \Rightarrow \frac{2762 - 2755.5}{1} x(0.4) + 2755.5 = 2758.1 kJ/kg$$

$\therefore h_3 = 2758.1$ kJ/kg

$$h_4 = h\Big|_{6.4 bar, 400^0 C}$$

From superheated steam tables, For P = 6.0 bar, h = 3270.6 s = 7.709

P = 7.0 bar, h = 3269.0 s = 7.6362

$\therefore$ For 6.4 bar, $h_4 \Rightarrow 3269.96$ kJ/kg

$S_4 \Rightarrow 7.6798$ kJ/kg-K

Process 4-5 is isentropic, $S_4 = S_5$

i.e., $7.6798 = 0.4763 + x_5 (7.9197)$

$\therefore x_5 = 0.909$

$\therefore h_5 = 137.8 + 0.909 (2423.8) = 2342.41$ kJ/kg

$$h_6 = h_f\Big|_{0.05 bar} = 137.8 kJ/kg$$

$h_7 = h_6$ (since W_{P1} is neglected)

$$h_8 = h_f\Big|_{6.4 bar} = 681.1 kJ/kg$$

$h_1 = h_8$ (since W_{P2} is neglected)

(ii) Applying energy balance to FWH,

$mh_3 + (1 - m)\, h_7 = h_8$

$m\,(2758.1) + (1 - m)\, 137.8 = 681.1$

$\therefore m = 0.313$ kg/kg of steam

(iii) $W_1 = W_{HP} = (h_2 - h_3) = (3398.8 - 2758.1)$

$= 640.7$ kJ/kg

$W_2 = W_{LP} = (1 - m)\,(h_4 - h_5)$

$= (1 - 0.313)\,(3269.96 - 2342.41)$

$= 637.2$ kJ/kg

$\therefore W_{net} = W_1 + W_2 = 1277.9$ kJ/kg

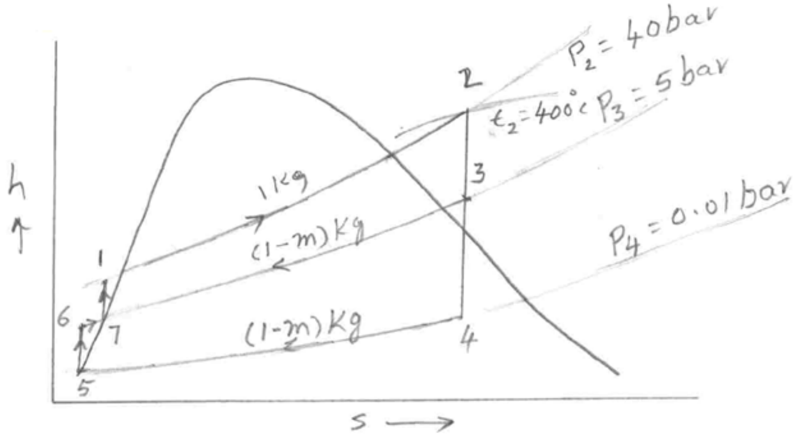
$$\therefore \text{Steam flow rate through HP turbine} = \frac{Power}{W_{net}} = \frac{80000}{1277.9} = 62.6\,kg/s$$

(iv) $\eta_{cycle} = ?$ $Q_H = (h_2 - h_1) + (1 - m)\,(h_4 - h_3) = 3069.35$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{1277.9}{3069.35} = 0.42$$

8. In a single heater regenerative cycle, the steam enters the turbine at 30 bar, 400^{0}C and the exhaust pressure is 0.01 bar. The feed water heater is a direct contact type which operates at 5 bar. Find (i) thermal η and the steam rate of the cycle, (ii) the increase in mean temperature of heat addition, η and steam rate as compared to the Rankine cycle without regeneration. Pump work may neglected.

Solution:



P_2 = 40 bar $t_2 = 400^0$C P_4 = 0.01 bar P_3 = 5 bar

From h-s diagram,

$$h_2 = h\Big|_{30\,bar,\,400^0 C} = 3230 kJ/kg$$

h_3 = 2790 kJ/kg

h_4 = 1930 kJ/kg

h_5 = 29.3 kJ/kg

h_7 = 640.1 kJ/kg

Since pump work may neglect, $h_6 = h_5$ & $h_1 = h_7$

(i) η_{cycle} = ?

Let m = mass of steam bled per kg boiler steam

Applying SFEE to FWH,

$mh_3 + (1 - m)\,h_6 = h_7$

m (2790) + (1 – m) 29.3 = 640.1

$\therefore$ m = 0.221 kg/kg of boiler steam

$W_T = (h_2 - h_3) + (1 - m)(h_3 - h_4)$

= (3230 – 2790) + (1 – 0.221) (2790 – 1930)

= 1109.73 kJ/kg

Q_H $= (h_2 - h_1) = (3230 - 640.1)$

$= 2589.9$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_T}{Q_H} = 0.428$$ Since W_P is neglected

$$(ii)\ steam\ rate = \frac{3600}{W_T} = 3.24 kg/kWh$$

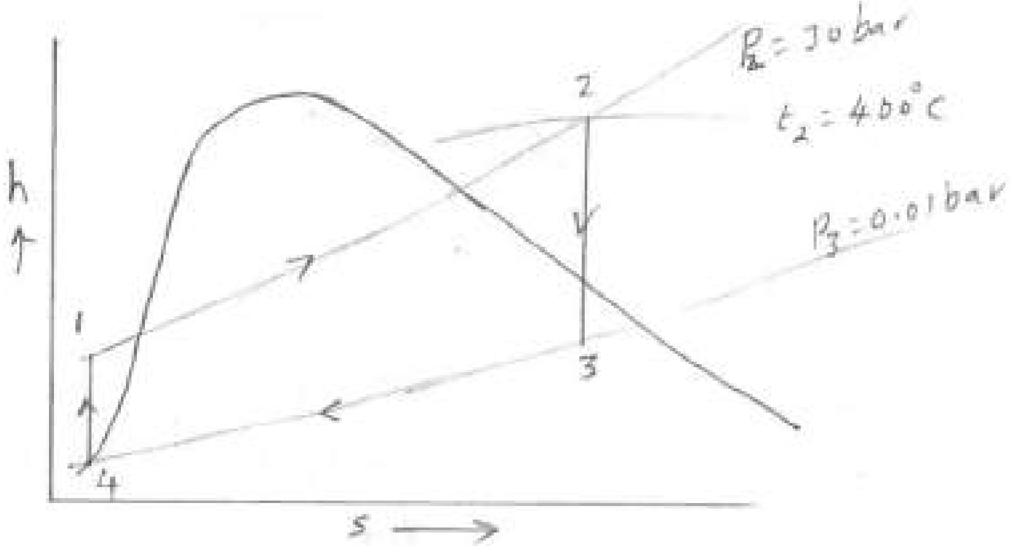
(iii) Mean temperature of heat addition, $\Delta T_m = \dfrac{Q_H}{s_2 - s_5}$

From h-s diagram, $s_2 = 6.83$ kJ/kg-K

From steam tables, $s_5 = 0.1060$ kJ/kg-K

$$\therefore \Delta T_m = \frac{2589.9}{(6.83 - 0.106)} = 385.2^0 K$$

Case (ii) Rankine cycle without Regeneration:



From h-s diagram,

$h_2 = 3230$ kJ/kg

$h_3 = 1930$ kJ/kg

$h_4 = 29.3$ kJ/kg

$h_1 = h_4$

$S_2 = 6.83$ kJ/kg-K

$S_4 = 0.1060$ kJ/kg-K

$$(i) \quad \eta_{cycle} = \frac{W_T}{Q_H} = \frac{(h_2 - h_3)}{(h_2 - h_1)}$$

$$= \frac{1300}{3200.7} = 0.41$$

$$(ii) \quad Steam\ rate = \frac{3600}{W_T} = 2.76 kg / kWh$$

(iii) Mean temperature of heat addition, $\Delta T_m = \frac{3200.7}{(6.83 - 0.106)} = 476^0 K$

Comparison	ΔT_m	η_{cycle}	Steam rate
Rankine cycle with regeneration	385.2 K	0.428	3.24 kg/kWh
Rankine cycle without regeneration	476^0K	0.41	2.76kg/kWh
$\therefore$ Increase w.r.t Rankine cycle	- 0.19 i.e., (-19%)	0.044 (4.4%)	0.174 (17.4%)

UNIT-1 COMBUSTION THERMODYNAMICS

1.1 Introduction

All conventional fossil fuels, whether, solid, liquid or gaseous, contain basically carbon and hydrogen which invariably react with the oxygen in the air forming carbon dioxide, carbon monoxide or water vapour. The heat energy released as a result of combustion can be utilized for heating purposes or for generation of high pressure steam in a boiler or as power from an engine or a gas turbine.

The solid fuels are burned in beds or in pulversied from suspended in the air stream. The liquid fuels are burned either by vaporising and mixing with air before ignition, when they behave like a gaseous fuel. The gaseous fuels are either burned in burners when the fuel and air are premixed or the fuel and air flow separately in to a burner or a furnace and simultaneously mix together as combustion proceeds.

The Kg-mole or gram-mole is widely used in combustion calculations as a unit of weight. The molecular weight of any substance in kg represents one kilogram mole or 1K mole. 1Kmol of hydrogen has a mass of 2.016Kg and 1Kmol of carbon has a mass of 12Kg.

Consider a reaction

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \qquad 1.1$$
$$(12+4.032) + 64 \rightarrow 44 + 2\,(2.032 + 16)$$

16.032kg of methane reacts with 64Kg of oxygen to form 44kg of carbon dioxide and 36.032kg of water. We can also simply state that 1Kmol of methane reacts with 2Kmol of oxygen to form 1Kmol of carbon dioxide and 2K mol of water, this has advantage of permitting easy conversion between the mass and volumetric quantities for the gaseous fuel and the product of combustion. If the gases are considered ideal then according to Avogadro hypothesis, all gases contain the same number of molecules per unit volume. It implies that 1K mole of any gaseous substance occupies the volume of 22.4m^3 at NTP i.e., 1.013bar and 273K.

$$CH_4 + 20_2 \rightarrow CO_2 + 2H_2O \qquad 1.2$$

1 volume of methane reacts with 2 volume of oxygen to form one volume of CO_2 and two volumes of H_2O. Therefore in any reactions, the mass in confirmed but the no. of mol or volumes may not be considered.

(i) $C + O_2 \rightarrow CO_2$

On mass basis $(12) + (32) \rightarrow (44)$

$1\text{ Mol }(C) + 1\text{ Mol }(O_2) \rightarrow 1\text{ Mol }(CO_2)$

(ii) $2C + O_2 \rightarrow 2CO$

On mass basis $(24) + (32) \rightarrow (56)$

Volume basis 2 Mol (C) + 1 Mol (O_2) → 2Mol (CO)

(iii) $2C + O_2 \rightarrow 2CO_2$

On mass basis $(56) + (32) \rightarrow (88)$

2 Mol (CO) + 1 Mol (O_2) → 2 Mol (CO_2)

1.2 Combustion Stoichometry

A balanced chemical equation for complete Combustion of the reactions with no excess air in the product is known as a stiochiometric equation. A stiochiometric mixture of the reactants is one in which the molar proportions of the reactants are exactly as given by the stiochiometric coefficients, so that no excess of any constituent is present. In general a chemical reaction may be written as

$$aA + bB \quad - \quad cC + dD \qquad 1.3$$

Where the reactants A and B react to form the products C and D. The small letters a, b, c and d are known as the stiochiometric coefficients.

For the combustion of any fuel the most common oxidizer is air which is a mixture of 21% O_2 and 79% N_2 (on volume basis). One mol of oxygen is accompanied by 79/21 (3.76) mol of Nitrogen. The Chemical equation for the stiochiometric combustion of carbon with air is written as

$$C + O_2 + 3.76N_2 \rightarrow CO_2 + 3.76N_2 \qquad 1.4$$

The minimum amount of air required for the complete combustion of a fuel is known as theoretical air. However in practice it is difficult to achieve complete combustion with theoretical air. Therefore fuel requires some excess air for different application and may vary from 5% ~ 20% and in gas turbine it may go up to 400% of theoretical quantity.

1.3 Theoretical air required for complete combustion.

If the fuel composition is known, the requirement of oxygen or air can be calculated either by mass balance or by mole method.

Consider a equation

$$C + O_2 \rightarrow CO_2 \qquad 1.5$$

$$(12) + (32) \rightarrow (44)$$

Or $1 + 8/3 \rightarrow 11/3$

Or 1 Kg C + 8/3 Kg O_2 → 11/3 kg CO_2

Similarly

$2C + O_2 \rightarrow 2CO$ 1.6
$2(12) + (32) \rightarrow 2(28)$
Or $1\ Kg\ C + 32/24\ Kg\ O_2 \rightarrow 56/24\ kg\ CO$
Or $1\ Kg\ C + 4/3\ Kg\ O_2 \rightarrow 7/3\ kg\ CO$

Similarly

$1\ Kg\ CO + 4/7\ Kg\ O_2 \rightarrow 11/7\ kg\ CO_2$ 1.7

On molal basis

$1K\ mol\ C + 1K\ mol\ O_2 \rightarrow 1K\ mol\ CO_2$
$1K\ mol\ C + ½\ K\ mol\ O_2 \rightarrow 1K\ mol\ CO$
$1K\ mol\ C + ½\ K\ mol\ O_2 \rightarrow 1K\ mol\ CO_2$

1.4 Conversion of Gravimetric analysis to volumetric basis and vice versa

If the composition of fuel is given on gravimetric (or weight) basis it can be converted to volumetric (or mole) basis as follows. Divide the weight of each constituents of the mixture by its molecular weight. This will give the relative volume (or mole) of each constituents. Add all the relative volumes of the constituents then,

$$\frac{\text{Indivisual (relative) volume of the constituents}}{\text{Total volume of all the constituents}} \times 100$$

will give the %age by volume of each constituents in the fuel.

If the volumetric composition of a fuel is given, it can be converted to gravimetric (or weight) basis as follows. Multiply the indivisual volume of each constituent by its molecular weight. This will give relative weight of each constituent. Add all the relative weights of the constituents then

$$\frac{\text{Indivisual weight of the constituents}}{\text{Total(reative)weights of the constituents}} \times 100$$

will give the %age by weight of each constituent in the fuel.

1.4.1 Calculation of the minimum amount of air for a fuel of known composition.

Example 1
Calculate the minimum volume of air required to burn one Kg of coal having the following composition by weight

C = 72.4%, H_2 5.3%, N_2 = 1.81, O2 = 8.5%, moisture 7.2%
S = 0.9% and ash 3.9%

On weight basis:

Taking 1kg coal as basis weight of oxygen required to burn 1kg of coal

$$C + O_2 \rightarrow CO_2$$

$0.724 \times 32/12 = 1.93$ kg

$0.53 \times 16/2 = 0.424$ kg

$0.009 \times 32/32 = \underline{0.009}$ kg

Total O_2 = 2.363 kg per kg of coal

But 0.085kg O_2 is available in coal, therefore O_2 required

= 2.363 – 0.085 = 2.278kg per Kg of coal.

Air contains 23% of oxygen by weight. Therefore the weight of the air supplied is

2.278x 100/23 = 9.9 kg per kg of coal

Density of air required at NTP

$P v = mRT$

$P = m/v\ RT = \rho RT$,

$$\rho = \frac{\text{Molecular weight}}{\text{Volume}}$$

$= P/RT = 1.013 \times 10^5/287 \times 273 = 1.29\ kg/m^3$

Therefore volume of air required = 9.9(kg)/1.29(kg) = 7.67 m^3

On mole basis

Consider 100kg of coal

C = 72.4/12 = 6.03K mol, O_2 = 8.5/32 = 0.265K mol
H_2 = 5.3/2 = 2.65K mol, H_2O = 7.2/18 = 0.4K mol

N_2 = 1.8/28 = 0.064K mol, S = 0.9/32 = 0.028K mol

1K mol C + 1K mol O_2 → 1Km CO_2

Therefore 6.03 K mol of carbon requires
6.03 K mol of oxygen

1 K mol H_2 + ½ K mol O_2 → 1K mol H_2O
H2 - 2.65 x ½ = 1.325K mol
S - 0.028x1 = 0.028
Total O_2 required 6.03 + 1.325 + 0.028 = 7.383

The oxygen present in coal 0.265K mol

Net O_2 required = 7.383 – 0.265 = 7.118K mol

Air required

7.118x100/21 = 33.89K mol / 100kg of coal = 0.3389K mol / 1kg coal

Volume of air supplied
0.3389K mol/kg X 22.4m^3 = 7.59m^3/kg of coal

Example 2

Calculate the volumetric analysis of the flue gases when coal burns with 20% excess air from the previous calculation the actual air required 33.89K mol/100kg coal. Therefore the actual air is
33.89 x 120/100 = 40.67K mol/ 100 kg coal

The amount of N_2 associated with this

40.67 x 79/100 = 32.13K mol

The amount of O_2 present 40.67 x 21/100 = 8.54K mol

The actual amount of O_2 required was 7.118K mol excess O_2 will appear in exhaust gas = 8.54 – 7.118 = 1.422K mol.

Therefore:
CO_2 = 6.03K mol
SO_2 = 0.028K mol
N_2 = 32.13K mol (air) + 0.064 (fuel)
= 32.194K mol
O_2 = 1.422K mol os excess oxygen.
Therefore the Total volume = (6.03 + 0.028 + 32.194 + 1.422)
= 39.674K mol

The volumetric composition of the gas

CO_2 = (6.03/39.674) x 100 = 15.12%

SO_2 = (0.028/39.674) x 100 = 0.07%

N_2 = (32.13/39.674) x 100 = 81.15%

O_2 = (1.422/39.674) x 100 = 3.58%

1.5 Calculation of the composition of fuel and excess air supplied from the exhaust gas analysis:

Some times the composition of fuel is unknown and it becomes necessary to judge whether the amount of air supplied is sufficient or not, or excess. This can be obtained by analyzing the sample of exhaust gases.

Example 3

The composition of dry flue gases obtained by burning a liquid fuel containing only hydrogen and carbon is CO_2 10.7%, O_2 5.1%, N_2 84.2%. Calculate the composition of fuel by weight and excess air used.

Solution: consider 100K mol of dry flue gases. They will contain 10.7K mol of O_2 (from CO_2) + 5.1K mole of (as max. oxygen) = 15.8K mol

Using nitrogen balance the actual air used 84.2 x 100/79 = 106.58K mol of dry flue gases and oxygen in the air supplied 106.58 x 21/100 = 22.38K mol. Therefore the amount of O_2 present in the water produced by the combustion of H_2 is 22.38 – 15.8 = 6.58K mol O_2. We know that 1 K mole of H_2 combines with ½ K mol O_2 to produce water. Therefore the amount of hydrogen present is 6.58x2 = 13.16K mol/100K mol of dry flue gases, and the carbon present is 12X10.7 = 128.4kg/100K mol of dry flue gas. Therefore the composition of fuel (by weight) is 128.4kgC and 26.32Kg H_2 on the %age basis.

C = (128.4/(128.4+26.32) x 100 = 82.99%

H = (26.32/(128.4+26.32) x 100 = 17.01%

Excess air supplied

The amount of O_2 required to burn 10.7K mol C is 10.7K mol and to burn 13.16K mol H_2 is 13.16 X ½ = 6.58

Total O_2 required = 10.7 + 6.58 = 17.28K mol/100K mol of dry flue gases

%age of excess air = (22.38 – 17.28)/(17.28) x 100 = 29.5%

1.6 Dew point of products:

The product of combustion containing water vapour are known as wet products. The water vapour present in combustion product is cooled down to a point of condensation the vapour turn in to liquid and volume will be reduced. Knowing the partial pressure exerted by the water before condensing, it is possible to find the saturation temp. corresponding to partial pressure from the steam tables.

1.7 Flue gas analysis

A fuel has the following %age volumetric analysis

H_2 = 48 CH_4 : 26, CO_2 : 11, CO : 5, N_2 = 10

The %age volumetric analysis of the dry exhaust gases in CO_2:8.8, O_2: 5.5, N_2: 85.7

Determine the air/fuel ratio by volume if air contains 21% O_2 by volume

Solution: the chemical equation for the reaction of 100 moles of fuel gas with air may be written as

$$48H_2 + 26CH_4 + 11CO_2 + 5CO + 10N_2 + x (O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

Carbon balance (C) $\rightarrow$ $26 + 11 + 5 = a = H_2$
H_2 $\rightarrow$ $48 + 52 = b = 100$
O_2 $\rightarrow$ $11 + 2.5 + x = a + c$ (i)
N_2 $\rightarrow$ $10 + 3.76x = d$ (ii)

Solving (i) and (ii) we, have

From (i) $11 + 2.5 + x = 100/2 + a + c$
$13.5 + x = 50 + a + c$
Adding $\underline{10 + 3.76x = d}$
$23.5 + 4.76x = 50 + a + c + d$
Or $a + c + d = 4.76x – 26.5$
% CO_2 by volume in dry gas
$(a/a+c+d) \times 100 = 8.8$
or $(42/4.76x – 26.5) = 0.088$
$4.76x = 503.77$

$$AF = \frac{\text{Total mol air}}{\text{Total mol fuel}} = \frac{503.77}{100} = 5.038\%$$

Example 4

A blast furnace gas has the following volumetric analysis H_2 CO-24%, CH_4 – 2%, CO_2-6%, O_2-3% and N_2-56%

Determine the Ultimate gravimetric analysis

Given volumetric analysis, H_2 – 9%, CO-24%, CH_4 – 2%, CO_2-6%, O_2-3% and N_2-56%

Solution: The volumetric analysis may be converted into mass or granite metric analysis by completing the table as follows:

Constituent	Volume in 1m3 of flue gas (a)	Molecular mass (b)	Proportional mass (c)=(a)x(b)	Mass in kg per kg of the gas (d)=(c)/ Σ ©	% by mass = (d)x100
CO	0.24	28	6.72	6.72/18.48 = 0.36	36%
CH_4	0.02	16	0.32	0.32/18.48 = 0.0173	1.73%
CO_2	0.06	44	2.64	264/18.48 = 0.142	14.2%
O_2	0.03	32	0.96	0.96/18.48 = 0.0519	5.19%
N_2	0.56	14	7.84	7.84/18.48 = 0.42	42%
			Σc = 18.48	Σ (d) = 1	100

The volumetric analysis of flue gas components becomes
CO-0.36, CH_4 – 0.0173, CO_2- 0.142, O_2-0.0519 and N_2-0.42

Example 5

Determine the fuel gas analysis and air fuel ratio by weight when fuel oil with 84.9% carbon, 11.4% hydrogen, 3.2% sulphur, 0.4% oxygen and 0.1% ash by weight is burnt with 20% excess air, assume complete combustion.

Solution: Consider 1kg of fuel

Oxygen required / Kg of fuel

For burning of 1kg C - 0.849 x32/12

For burning of 1kg H - 0.114 x16/2

For burning of 1kg S - 0.032 x32/32

Total O_2 required is 3.208 kg.

Amount of O_2 contained in the fuel = 0.004Kg

Net O_2 supplied / kg of fuel = 3.208 – 0.004

= 3.204 kg O_2

Net air supplied = 3.204x100/23 = 13.93 kg/kg of fuel

When 20% excess air supplied

Total air supplied = 13.93 x 1.2 = 16.716 kg/kg of fuel.

N_2 actually supplied = 16.716 x 77/100 = 12.871 kg/kg of fuel

O_2 actually supplied = 16.716 x 23/100 = 3.845 kg/kg of fuel

Total free O_2 in fuel gas = 3.845 – 6.204

= 0.641 kg/kg of fuel

Total free N_2 in fuel gas = 12.87 kg/kg of fuel

Flue gas analysis:

C converted to CO_2 = 0.849x44/12 = 3.113 kg CO_2

H converted to H_2O = 0.114x18/2 = 1.026 kg H_2O

S converted to SO_2 = 0.032x64/32 = 0.064 kg SO_2

Flue gas / kg of fuel:

$$= \underset{CO_2}{3.113} + \underset{H_2O}{1.26} + \underset{SO_2}{0.064} + \underset{O_2}{0.641} + \underset{N_2}{12.871}$$

$$= 17.715\text{kg.}$$

Therefore:

CO_2 = (3.113/17.715)x100 = 17.573%

SO_2 = (0.064/17.715)x100 = 0.36%

O_2 = (0.641/17.715)x100 = 3.618%

H_2O = (1.026/17.715)x100 = 5.79%

N_2 = (12.871/17.715)x100 = 72.656%

Air fuel mixture ratio is = 16.716 : 1

Example 6

A blast furnace gas has the following volumetric analysis.

H_2 = 9%, CO = 24%, CH_4 = 2%, CO_2 = 6%, O_2 = 3% and N_2 = 56 %

Determine the ultimate gravimetric analysis.

Solution:

Total H2 in the blast furnace gas.

% volumetric analysis = $9H_2 + 2H_4$

Proportional mass = % volumetric analysis X mol. Mass of element

= (9x2) + (2x4) = 18 + 8 = 26 kg.

Total ‘C’ in the blast furnace gas.

% of volumetric analysis = 24C + 2C + 6C

Proportional mass = (24+2+6) x 12

= 384 kg

Total O_2 in the blast furnace gas

% of volumetric analysis = 24xO + $6O_2$ + $3O_2$

Proportional mass = (24+16) x 9 (32)

= 672 kg

Total N_2 in the blast furnace gas

% of volumetric analysis = 56 N_2

Proportional mass of N_2 = 56 x 28 = 1568 Kg.

Total weight of blast furnace gas:

= 384kg C + 26kg H_2 + 672kg O_2 + 1568kg N_2
= 2650kgs

Gravimetric %age compositon:

C = (384/2650)x100 = 14.49%

H_2 = (26/2650)x100 = 0.98%

O_2 = (672/2650)x100 = 25.36%

N_2 = (1568/2650)x100 = 59.17%

Example 7

The analysis of coal used in a boiler trial is as follows. 82% carbon, 6% hydrogen, 4% oxygen, 2% moisture and 8% ash. Determine the theoretical air required for complete combustion of 1kg of coal. If the actual air supplied is 18kg per kg of coal the hydrogen is completely burned & 80% carbon burned to CO_2 ,the reminder is CO, Determine the volumetric analysis of the dry products of combustion.

Solution: For complete combustion.

O_2 required is

For carbon - 0.82 = 2.186 kg of O_2

For hydrogen - 0.006 = 0.48 kg of O_2

Total O_2 required = 2.666kg.

Net O_2 supplied = Total O_2 required – O_2 present in the fuel

= 2.66 – 0.004

= 2.662 kg/kg of coal

Theoretical minimum air required for complete combustion [C burns to CO_2 totally]

Air supplied = 2.626x100/23 = 11.417 kg/kg of coal

Flue gas analysis:

But actually only 80% carbon is burns to CO_2

CO_2 = 0.8 x 0.82 x44/12 = 2.405kg of CO_2

20% carbon is burnt to CO

CO = 0.2 x 0.82 x 28/12 = 0.383 kg of CO

O_2 actually required for 80% carbon burnt to CO_2

= 0.8 x 0.82 x 3232/12 = 1.749 kg of O_2

O_2 actually required for 20% carbon burnt to CO

= 0.2 x 0.82 x 16/12 = 0.219 kg of O_2

O_2 required by Hydrogen:

= 0.06 x 8 = 0.48 kg of O_2.

H_2O produced = 0.06 x 9 = 0.54 kg of H_2O

But actual air supplied = 18kg

Actually O_2 supplied = 18 x23/100 = 4.14 kg of O_2

Free O_2 in the flue gas = 4.14 + 0.04 – 1.749 – 0.219 – 0.48

= 1.732 kg of O_2/kg of coal

N_2 in the flue gas = 18x77/100 = 13.86 kg/kg of coal

Volumetric analysis of the dry products of combustion.

CO_2 = (2.405/44)x 100 = 0.0546m /K. mol

CO = (0.383/28) x 100 = 0.0137 m^3/K.mol

O_2 = (1.732/32)x 100 = 0.0541m^3/K.mol

N_2 = (13.86/28)x 100 = 0.495m^3/K.mol

In % of volume:

CO_2 = (0.0546/0.6174)x 100 = 8.84%

CO = (0.0137/0.6174)x 100 = 2.22%

O_2 = (0.0541/0.6174)x 100 = 8.76%

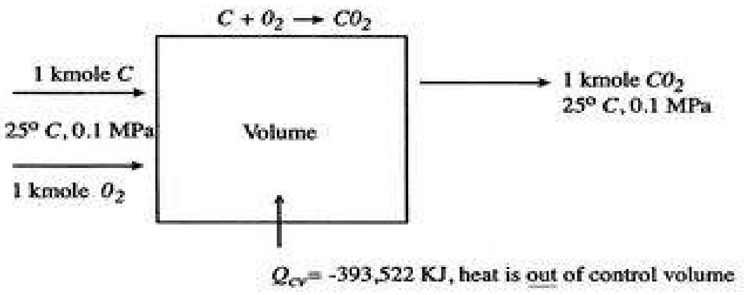
N_2 = (0.495/0.6174)x 100 = 80.70%

1.8 Enthalpy of reaction

Enthalpy of a reaction is defined as the difference between the enthalpy of the products at a specified state and the enthalpy of the reactants at the same state for a complete reaction. For combustion process, the enthalpy of a reaction is usually referred to as the “enthalpy of combustion” it is obviously a very useful property for analyzing the combustion processes of fuels. However there are so many different fuels and fuel mixtures that is not practical to list enthalpy of combustion values for all possible cases. Besides, the enthalpy of combustion is not of much use when the combustion is incomplete. Therefore a more practical approach would be have a more fundamentally property to represent the chemical energy of an element or compound at some reference state. This property is the “enthalpy of formation” which can be viewed as the enthalpy of a substance at a specified state due to its chemical composition. To establish a starting point it is assigned the enthalpy of formation for all stable elements such as $O_{2,}$ N_2, H_2 and C a value of zero at standard reference state of 25°C and 1 atm. For all stable compounds.

In a chemical reaction bonds are broken in the reactants and new bonds formed in the products. Energy is required to break bonds and energy is released when bonds are formed. The energy associated with a chemical reaction depends on the number and type of bonds broken and/or formed.

Every chemical species has a certain amount of "heat content," or enthalpy, H, which cannot be measured. However, differences in enthalpy can be measured. The net energy change for a reaction performed at constant pressure is the enthalpy change for the reaction. This enthalpy change, ΔH, has units kJ/mol and is defined:



[C + H (fuel)] + [O_2 + N_2 (Air)] -> (Combustion Process) -> [CO_2 + H_2O + N_2 (Heat)]

where

C = Carbon, H = Hydrogen, O = Oxygen, N = Nitrogen

(1) $\Delta H = H(\text{products}) - H(\text{reactants})$

If energy is given off during a reaction, such as in the burning of a fuel, the products have less heat content than the reactants and ΔH will have a negative value; the reaction is said to be exothermic. If energy is consumed during a reaction, ΔH will have a positive value; the reaction is said to be endothermic.

The enthalpy change for a chemical change is independent of the method or path by which the change is carried out as long as the initial and final substances are brought to the same temperature. This observation, known as HESS'S LAW, has important practical utility.

Thermochemical equations may be treated as algebraic equations: they may be written in the reverse direction with a change in the sign of ΔH – even though the reverse reaction may not actually occur; they may be added and subtracted algebraically; the equation and associated ΔH value may be multiplied or divided by factors. Hess's Law allows the calculation of enthalpy changes that would be difficult or impossible to determine directly, i.e. by experiment.

The enthalpy change for the reaction:

(2) $2C\ (s) + O_2\ (g) \rightarrow 2CO\ (g)$

cannot be determined directly because carbon dioxide will also form. However, □H can be measured for:

(3) $C\ (s) + O_2\ (g) \rightarrow CO_2\ (g)$ $\Delta H = -393.5$ kJ

(4) $2CO\ (g) + O_2\ (g) \rightarrow 2CO_2\ (g)$ $\Delta H = -566.0$ kJ

Multiplying equation (3) by 2 gives equation (5), and reversing equation (4) gives equation (6):

(5) $2C\ (s) + 2O_2\ (g) \rightarrow 2CO_2\ (g)$ $\Delta H = -787.0$ kJ

(6) $2CO_2\ (g) \leftrightarrow 2CO\ (g) + O_2\ (g)$ $\Delta H = +566.0$ kJ

Adding equations (5) and (6) gives the desired information:

(2) $2C\ (s) + O_2\ (g) \rightarrow 2CO\ (g)$ $\Delta H = -221.0$ kJ

For a reaction in which a compound is formed from the elements, the enthalpy change is called the heat of formation, ΔH_f^o, for the compound. The superscript "o" indicates standard conditions of one atmosphere pressure. Equation (2) and (3) are such reactions. Some others:

(7) $S\ (s) + O_2\ (g) \rightarrow SO_2\ (g)$ $\Delta H = -296.9$ kJ

(8) $Mg\ (s) + Cl_2\ (g) \rightarrow MgCl_2\ (s)$ $\Delta H = -641.8$ kJ

In reactions (2), (3), (7), and (8) ΔH for the reaction is ΔH_f^o for the compound. For the reaction:

(9) $2S\ (s) + 3O_2\ (g) \rightarrow 2SO_3\ (g)$ $\Delta H = -790.4$ kJ

the heat of reaction is associated with the formation of two moles of SO_3. But heat of formation is per mole of compound, so ΔH_f^o for SO_3 is half of –790.4, or –395.2 kJ.

Extensive listings of heats of formation are available in handbooks. With these values of ΔH_f^o, you can calculate virtually any heat of reaction. The heat of a reaction is the sum of ΔH_f^o values for the products minus the sum of ΔH_f^o values for the reactants. Expressed as a formula:

(10) $\Delta H_{rxn} = \Sigma\Delta H_f^{\circ}{}_{products} - \Sigma\Delta H_f^{\circ}{}_{reactants}$

Heats of formation for several compounds are given below. Note that the phase of the compound is important when choosing a ΔH_f for a free element is zero.

STANDARD HEATS OF FORMATION, ΔH_f^o, kJ/mole, at 25°C

AgCl (s)	–127.1	$Ca(OH)_2$ (s)	–986.1	K_3PO_4 (aq)	–2002.9
$AgNO_3$ (aq)	–100.7	$Ca(OH)_2$ (aq)	–1002.9	K_2SO_4 (aq)	–1409.2
$AlCl_3$ (s)	–695.4	HCl (g)	–92.3	$MgCl_2$ (aq)	–797.1
$AlCl_3$ (aq)	–1027.2	HCl (aq)	–167.4	$Mg(NO_3)_2$ (aq)	–875.1
$Al(OH)_3$ (s)	–1272.8	H_2O (g)	–241.8	NaCl (aq)	–407.1
$Al_2(SO_4)_3$ (aq)	–3753.5	H_2O (l)	–285.8	$NaHCO_3$ (s)	–947.7
$BaCl_2$ (aq)	–873.2	H_3PO_4 (aq)	–1294.1	$NaNO_3$ (aq)	–446.2
$Ba(NO_3)_2$ (aq)	–951.4	H_2SO_4 (l)	–814.0	NaOH (aq)	–469.4
$BaSO_4$ (s)	–1473.2	H_2SO_4 (aq)	–888.0	Na_2SO_4 (aq)	–1387.0
$CaCl_2$ (aq)	–877.8	KOH (aq)	–481.2	$ZnCl_2$ (aq)	–487.4

EXAMPLE: Using ΔH_f^o data calculate the heat of reaction for:

(11) $AgNO_3\text{ (aq)} + NaCl\text{ (aq)} \rightarrow AgCl\text{ (s)} + NaNO_3\text{ (aq)}$

$$\Delta H^o = [\Delta H_f^o\, AgCl\text{ (s)} + \Delta H_f^o\, NaNO_3\text{ (aq)}] - [\Delta H_f^o\, AgNO_3\text{ (aq)} + \Delta H_f^o\, NaCl\text{ (aq)}]$$

$$= [(-127.0) + (-446.2)] - [(-100.7) + (-407.1)]$$

$$= [-573.2] - [-507.8] = -573.2 + 507.8 = -65.4\text{ kJ}$$

EXAMPLE Using ΔH_f^o data calculate the heat of reaction for

(12) $2\, AgNO_3\text{ (aq)} + MgCl_2\text{ (aq)} \rightarrow 2\, AgCl\text{ (s)} + Mg(NO_3)_2\text{ (aq)}$

$$\Delta H^o = [2\, \Delta H_f^o\, AgCl\text{ (s)} + \Delta H_f^o\, Mg(NO_3)_2\text{ (aq)}] - [2\, \Delta H_f^o\, AgNO_3\text{ (aq)} + \Delta H_f^o\, MgCl_2\text{ (aq)}]$$

$$= [2(-127.0) + (-875.1)] - [2(-100.7) + (-797.1)]$$

$$= [-1129.1] - [-998.5] = -1129.1 + 998.5 = -130.6\text{ kJ}$$

Note: the values of ΔH_f^o
chemical equation.

are multiplied by the stiochiometric coefficients from the balanced

(13) $2C\ (s) + O_2\ (g) \rightarrow 2CO\ (g)$ $\Delta H = -221.0$ kJ

(14) $C\ (s) + O_2\ (g) \rightarrow CO_2\ (g)$ $\Delta H = -393.5$ kJ

(15) $2H_2\ (g) + O_2 \rightarrow 2H_2O\ (g)$ $\Delta H = -483.6$ kJ

(16) $2H_2\ (g) + O_2 \rightarrow 2H_2O\ (l)$ $\Delta H = -571.8$ kJ

(17) $CaO\ (s) + CO_2\ (g) \rightarrow CaCO_3\ (s)$ $\Delta H = -178.1$ kJ

(18) $2Ca\ (s) + O_2\ (g) \rightarrow 2CaO\ (g)$ $\Delta H = -1271.0$ kJ

Using Hess' Law with appropriate equations from (13)-(18), above, calculate □H for each of the following reactions:

1) $H_2O\ (l) \rightarrow H_2O\ (g)$

2) $C\ (s) + H_2O\ (g) \rightarrow CO\ (g) + H_2\ (g)$

3) $Ca\ (s) + H_2O\ (g) \rightarrow CaO\ (s) + H_2\ (g)$

4) $CO\ (g) + 1/2\ O_2\ (g) \rightarrow CO_2\ (g)$

5) $2Ca\ (s) + 2C\ (s) + 3O_2\ (g) \rightarrow 2CaCO_3\ (s)$

Using heats of formation values from page T-56 calculate □H for each of the following reactions:

6) $2Al\ (s) + 3Cl_2\ (g) \rightarrow 2AlCl_3\ (s)$

7) $2Al\ (s) + 3H_2SO_4\ (aq) \rightarrow Al_2(SO_4)_3\ (aq) + 3H_2\ (g)$

8) $2Al\ (s) + 3ZnCl_2\ (aq) \rightarrow 2AlCl_3\ (aq) + 3Zn\ (s)$

9) $3BaCl_2\ (aq) + Al_2(SO_4)_3\ (aq) \rightarrow 3BaSO_4\ (s) + 2AlCl_3\ (aq)$

10) $Na_2SO_4\ (aq) + BaCl_2\ (aq) \rightarrow BaSO_4\ (s) + 2NaCl\ (aq)$

11) $BaCl_2\ (aq) + 2AgNO_3\ (aq) \rightarrow 2AgCl\ (s) + Ba(NO_3)_2\ (aq)$

12) $Ca(OH)_2\ (aq) + 2HCl\ (aq) \rightarrow CaCl_2\ (aq) + 2H_2O\ (l)$

13) $2Al(OH)_3\ (s) + 3H_2SO_4\ (aq) \rightarrow Al_2(SO_4)_3\ (aq) + 6H_2O\ (l)$

14) $AlCl_3\ (aq) + 3NaOH\ (aq) \rightarrow Al(OH)_3\ (s) + 3NaCl\ (aq)$

15) $3KOH\ (aq) + H_3PO_4\ (aq) \rightarrow K_3PO_4\ (aq) + 3H_2O\ (l)$

Answers to Problems

1) +44.1 kJ
2) +131.3 kJ
3) –393.7 kJ
4) –283.0 kJ
5) –2414.2 kJ
6) –1390.8 kJ
7) –1089.5 kJ
8) –592.2 kJ
9) –76.9 kJ
10) –19.2 kJ
11) –130.8
12) –111.7
13) –258.7
14) –58.7 kJ
15) –122.6

1.9 Internal Energy of Combustion: It is defined as the difference between the internal energy of the products and the internal energy of the reactants when complete combustion occurs at a given temperature and pressure.

$$U_c = U_p - U_R$$

$$= \quad \Sigma_p\, n_e\, (h_f + \Delta h - pv) - \Sigma_R\, n_i\, (h_f + \Delta h - pv)$$

1.10 Combustion efficiency
It is defined as the ratio of ideal fuel-air to the actual fuel-air ratio

$$\eta_{comb} = \frac{(F/A)_{ideal}}{(F/A)_{actl}}$$

Example 8
Consider the following reaction, which occurs in a steady state, steady flow processes.

$$CH_4 + 2O_2 \quad \rightarrow \; CO_2 + 2H_2O\ (l)$$

The reactants and products are each at total pressure of 0.1Mpa and 25^oC. Determine the heat transfer for per K mol of fuel entering the combustion chamber.

Solution : using the values of enthalpy of formation

$$Q = \quad h_f = \Sigma_p\, n_e h_f \;-\; \Sigma_R\, n_i h_f$$

$\Sigma_R\, n_i h_f = (h_f)\ CH_4 = -74873KJ$
$\Sigma_p\, n_e h_f = (h_f)\ CO_2 + 2\ (h_f)_{\ H2O\ (l)}$
$\qquad = -\ 393522+2\ (-2852830) = -\ 965182KJ$

Therefore Q = - 965182 – (-74873) = - 890309KJ

Example 9

A small gas turbine uses C8H18 ® as fuel and 400% theoretical air. The air and fuel enters at 25^oC and the products of combustion leaves at 900K. The output of the engine and the fuel consumption are measured and it is found that the specific fuel consumption is 0.25kg/Sec of fuel per MW out put. Determine the heat transfer from the engine per K mol of fuel. Assume complete combustion

Solution:

The combustion equation is

$$C_8H_{18} + 4\,(12.5)O_2 + 4\,(12.5)\,(\,3.76)N_2 \quad - \quad 8CO_2 + 9H_2O + 37.5O_2 + 188N_2$$

By first law

$$Q + \Sigma_R\, n_i\,(h_f + \Delta h); = W + \Sigma_p\, n_e\,(h_f + \Delta h)$$

$\Sigma_R\, n_i\,(h_f + \Delta h) = (h_f)_{C8H18} =$ 250105KJ/K mol fuel at 25^oC

Considering the products

$$\Sigma_p\, n_e\,(h_f + \Delta h) = nCO_2\,(h_f + \Delta h)_{CO2} + nH_2O\,(h_f + \Delta h)_{H2O} + nO_2\,(\Delta h)_{O_2} + nN_2\,(\Delta h)_{N_2}$$

h_f of O_2, N_2 = O Δh = Enthalpy of formation from 298^{o}K to 900K

Therefore $\Sigma_p\, n_e\,(h_f + \Delta h) = 8\,(-\,393522+288030)+9(-241826+21892)$
$+37.5(19249)+188(18222) = -755769$KJ/K mol fuel.

$$W = \frac{1000(KW)}{0.25} \quad X \quad \frac{114\ Kg}{K\ mol} = 456920KJ/K\ mol$$

Therefore $\qquad Q = -755769+456920-(-250105) = -48744$KJ/K mol fuel

CHAPTER 1

Testing of I.C.Engines

1.1. Introduction: - The basic task in the design and development of I.C.Engines is to reduce the cost of production and improve the efficiency and power output. In order to achieve the above task, the engineer has to compare the engine developed by him with other engines in terms of its output and efficiency. Hence he has to test the engine and make measurements of relevant parameters that reflect the performance of the engine. In general the nature and number of tests to be carried out depend on a large number of factors. In this chapter only certain basic as well as important measurements and tests are described.

1.2. Important Performance Parameters of I.C.Engines:- The important performance parameters of I.C. engines are as follows:

(i) Friction Power,

(ii) Indicated Power,

(iii) Brake Power,

(iv) Specific Fuel Consumption,

(v) Air – Fuel ratio

(vi) Thermal Efficiency

(vii) Mechanical Efficiency,

(viii) Volumetric Efficiency,

(ix) Exhaust gas emissions,

(x) Noise

1.3. Measurement of Performance Parameters in a Laboratory

1.3.1. Measurement of Friction Power:- Friction power includes the frictional losses and the pumping losses. During suction and exhaust strokes the piston must move against a gaseous pressure and power required to do this is called the "pumping losses". The friction loss is made up of the energy loss due to friction between the piston and cylinder walls, piston rings and cylinder walls, and between the crank shaft and camshaft and their bearings, as well as by the loss incurred by driving the essential accessories, such as water pump, ignition unit etc.

Following methods are used in the laboratory to measure friction power:

(i) Willan's line method;

(ii) From the measurement of indicated power and brake power;

(iii) Motoring test;

(iv) Retardation test;

(v) Morse Test.

1.3.1.1. Willan's Line Method:- This method is also known as fuel rate extrapolation method. In this method a graph of fuel consumption (vertical axis) versus brake power (horizontal axis) is drawn and it is extrapolated on the negative axis of brake power (see Fig. 1).The intercept of the negative axis is taken as the friction power of the engine at that speed. As shown in the figure, in most of the power range the relation between the fuel consumption and brake power is linear when speed of the engine is held constant and this permits extrapolation. Further when the engine does not develop power, i.e. brake

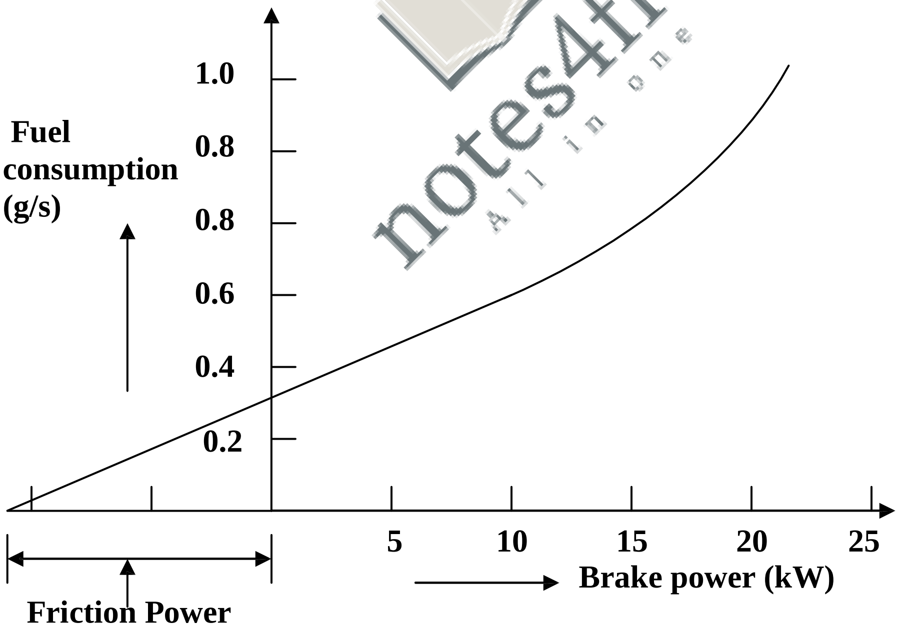


Figure.1 Willan's line method

power = 0, it consumes a certain amount of fuel. This energy in the fuel would have been spent in overcoming the friction. Hence the extrapolated negative intercept of the horizontal axis will be the work representing the combined losses due to friction, pumping and as a whole is termed as the frictional loss of the engine. This method of measuring friction power will hold good only for a particular speed and is applicable mainly for compression ignition engines.

The main draw back of this method is the long distance to be extrapolated from data between 5 and 40 % load towards the zero line of the fuel input. The directional margin of error is rather wide because the graph is not exactly linear.

1.3.1.2.From the Measurement of Indicated Power and Brake Power:- This is an ideal method by which friction power is obtained by computing the difference between the indicated power and brake power. The indicated power is obtained from an indicator diagram and brake power is obtained by a brake dynamometer. This method requires elaborate equipment to obtain accurate indicator diagrams at high speeds.

1.3.1.3.Morse Test:- This method can be used only for multi – cylinder IC engines. The Morse test consists of obtaining indicated power of the engine without any elaborate equipment. The test consists of making, in turn, each cylinder of the engine inoperative and noting the reduction in brake power developed. In a petrol engine (gasoline engine), each cylinder is rendered inoperative by *"shorting"* the spark plug of the cylinder to be made inoperative. In a Diesel engine, a particular cylinder is made inoperative by cutting off the supply of fuel. It is assumed that pumping and friction are the same when the cylinder is inoperative as well as during firing.

In this test, the engine is first run at the required speed and the brake power is measured. Next, one cylinder is cut off by short circuiting the spark plug if it is a petrol engine or by cutting of the fuel supply if it is a diesel engine. Since one of the cylinders is cut of from producing power, the speed of the engine will change. The engine speed is brought to its original value by reducing the load on the engine. This will ensure that the frictional power is the same.

If there are *k* cylinders, then

Total indicated power
when all the cylinders are working $= ip_1 + ip_2 + ip_3 + \ldots\ldots\ldots\ldots\ldots + ip_k = \sum_{j=1}^{k} ip_j$

We can write $\sum_{j=1}^{k} ip_j = B_t + F_t$ ……………………………………………..(1)

where ip_j is the indicated power produced by j th cylinder, k is the number of cylinders,

B_t is the total brake power when all the cylinders are producing power and F_t is the total frictional power for the entire engine.

If the first cylinder is cut – off, then it will not produce any power, but it will have frictional losses. Then

we can write $$\sum_{j=2}^{k} ip_j = B_1 - F_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

where B_1 = total brake power when cylinder 1 is cut - off and

F_t = Total frictional power.

Subtracting Eq. (2) from Eq. (1) we have the indicated power of the cut off cylinder. Thus

$$ip_1 = B_t – B_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3).$$

Similarly we can find the indicated power of all the cylinders, viz., ip_2, ip_3, …..$ip_{k.}$ Then the total indicated power is calculated as

$$(ip)_{total} = \sum_{j=1}^{k} ip_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4)$$

The frictional power of the engine is therefore given by

$$F_t = (ip)_{total} – B_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(5)$$

The procedure is illustrated by some examples worked out at the end of the chapter.

1.4. MEASUREMENT OF INDICATED POWER

The power developed in the cylinder is known as Indicated Horse Power and is designated as IP.

The IP of an engine at a particular running condition is obtained from the indicator diagram. The indicator diagram is the *p-v* diagram for one cycle at that load drawn with the help of indicator fitted on the engine. The construction and use of mechanical indicator for obtaining *p-v* diagram is already explained.

A typical *p-v* diagram taken by a mechanical indicator is shown in Figure 2.

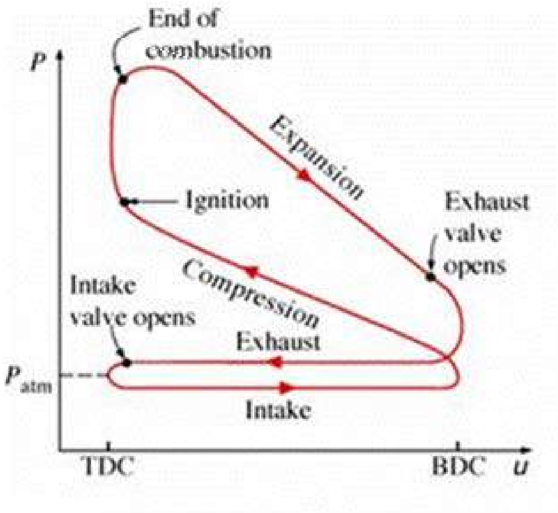


Figure.2 p-v diagram taken by mechanical indicator

The areas, the positive loop and negative loop, are measured with the help of a planimeter and let these be A_p and A_n cm^2 respectively, the net positive area is $(A_p – A_n)$. Let the actual length of the diagram as measured be L cm, then the average height of the net posiive area is given by

$$h=(A_p-A_n)/L \quad \text{in centimetre}$$

The height multiplied by spring-strength (or spring number) gives the indicated mean effective pressure of the cycle.

$$Imep=(A_p-A_n)*S/L \qquad .....(6)$$

Where S is spring scale and it is defined as a force per unit area required to compress the spring through a height of one centimeter (N/m^2/cm).

Generally the area of negative loop A_n is negligible compared with the positive loop and it cannot be easily measured especially when it is taken with the spring used for taking positive loop. Special light springs are used to obtain the negative loop. When two different springs are used for taking the *p-v* diagram of positive and negative loop, then the net indicated mean effective pressure is given by

$$P_m=A_p*S_p/L-A_n*S_n/L \quad .....(7)$$

Where S_p = Spring strength used for taking *p-v* diagram of positive loop, (N/m^2 per cm)
S_n = Spring strength used for taking *p-v* diagram of negative loop, (N/m^2 per cm)
A_p = Area in Cm2 of positive loop taken with spring of strength S_p
A_n = Area in Cm2 of positive loop taken with spring of strength S_n
Sometimes spring strength is also noted as spring constant.

The IP developed by the engine is given by

$$IP=P_mLAn/L \quad .....(8)$$

Where ‘n’ is the number of working strokes per second.

The explanation of this expression is already given in the last chapter.

1.5. MEASUREMENT OF B.P

Part of the power developed in the engine cylinder is used to overcome the internal friction. The net power available at the shaft is known as brake power and it is denoted by B.P. The arrangement used for measuring the BP of the engine is described below:

(a) Prony Brake. The arrangement of the braking system is shown in Figure 3. It consists of brake shoes made of wood and these are clamped on to the rim of the brake wheel by means of the bolts. The pressure on the rim is adjusted with the help of nut and springs as shown in Fig 2. A load bar extends from top of the brake and a load carrier is attached to the end of the load bar. Weight kept on this load carrier is balanced by the torque reaction in the shoes. The load arm is kept horizontal to keep the arm length constant.

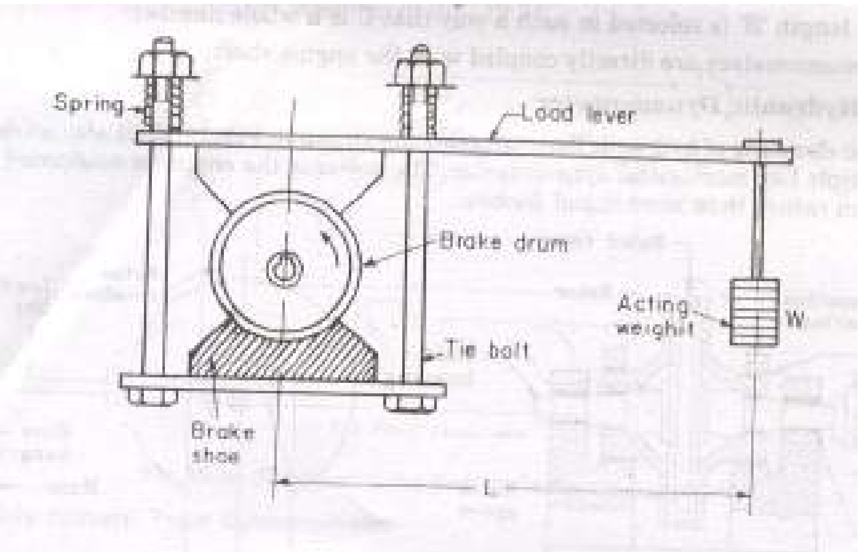


Fig. Prony-brake arrangement.

Figure.3

The energy supplied by engine to the brake is eventually dissipated as heat. Therefore, most of the brakes are provided with a means of supply of cooling water to the inside rim of the brake drum.

The BP of the engine is given by

B.P (brake power) = $2*\pi*N*T/60$ watts $=2*\pi*N*T/60*1000$ Kw ….. (9)

Where T = $(W.L)$ (N-m)

Where W = Weight on load carrier, (N)

And L = Distance from the centre of shaft to the point of load-meter in meters.

The prony brake is inexpensive, simple in operation and easy to construct. It is, therefore, used extensively for testing of low speed engines. At high speeds, grabbing and chattering of the band occur and lead to difficulty in maintaining constant load. The main disadvantage of the prony brake is its constant torque at any one band pressure and therefore its inability to compensate for varying conditions.

1.5.1 Hydraulic Dynamometer.

The BP of an engine coupled to the dynamometer is given by

B.P (brake power) = $2*\pi*N*W*R/60*1000 = WN(2*\pi*R/60*1000)$ Kw
The working of a prony brake dynamometer is shown in figure 4

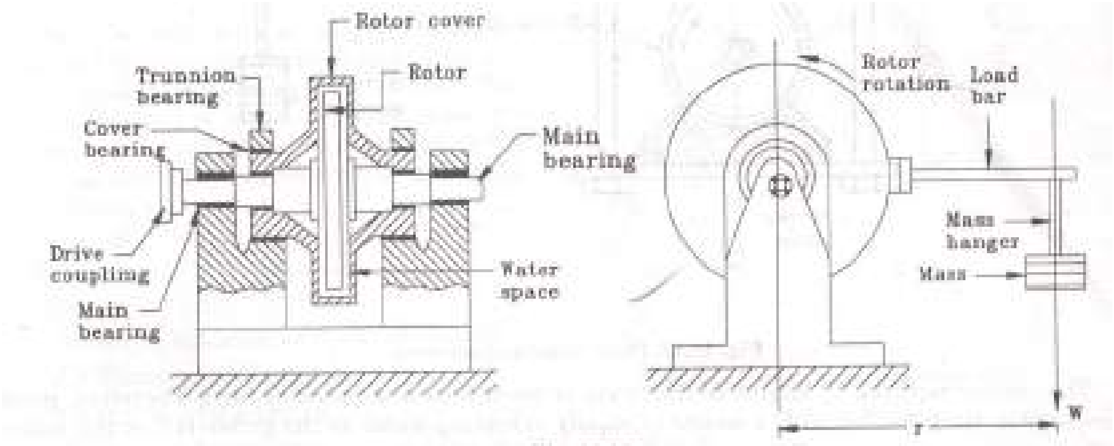


Figure.4 Hydraulic dynamometer

In the hydraulic dynamometer, as the arm length (R) is fixed, the factor [2 R/(60x1000)] is constant and its value is generally given on the name plate of the dynamometer by the manufacturer and is known as brake or dynamometer constant. Then the BP measured by the dynamometer is given by

$$B.P = \frac{WN}{K} \qquad …..(10)$$

Where W = Weight measured on the dynamometer, N
K = Dynamometer constant (60*1000/2*pi*R)
and N = RPM of the engine.

The arm length 'R' is selected in such a way that K is a whole number.
These dynamometers are directly coupled with the engine shaft.

1.5.2 Electric Dynamometer:

The electric generator can also be used for measured BP of the engine. The output of the generator must be measured by electrical instruments and corrected for generator

efficiency. Since the efficiency of the generator depends upon load, speed and temperature, this device is rather inconvenient to use in the laboratory for obtaining precise measurement. To overcome these difficulties, the generator stator may be supported in ball bearing trunnions and the reaction force exerted on the stator of the generator may be measured by a suitable balance. The tendency to rotate or the reaction of the stator will be equal and opposite to the torque exerted on the armature, which is driven by the engine which is shown in
Figure 5.

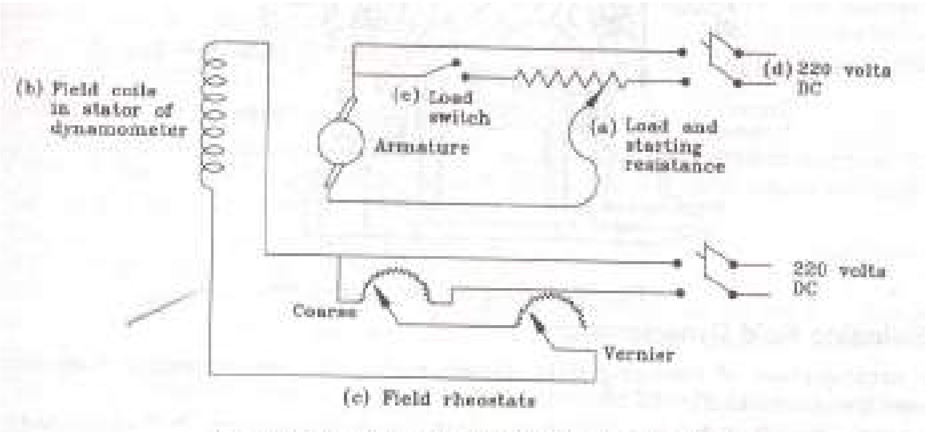


Fig. Simplified wiring diagram for electric dynamometer.

Figure.5

The electric dynamometer can also used as a motor to start and drive, the engine at various speeds.

There are other types of dynamometers like eddy current dynamometer, fan brake and transmission dynamometers used for measurement of large power output.

1.5.3 Eddy current Type Dynamomter

The ‘eddy-current’ dynamometer is an effect, a magnetic brake in which a toothed steel rotor turns between the poles of an electromagnet attached to a trunioned stator. The resistance to rotation is controlled by varying the current through the coils and hence, the strength of the magnetic field. The flux tends to follow the smaller air gaps at the ends of the rotor teeth and eddy currents are set up within the metal of the pole pieces, resulting in heating the stator. The heat energy is removed by circulating water through a water jacket formed in the stator. Figure 6 shows the “Heenan eddy-current dynamometer”.

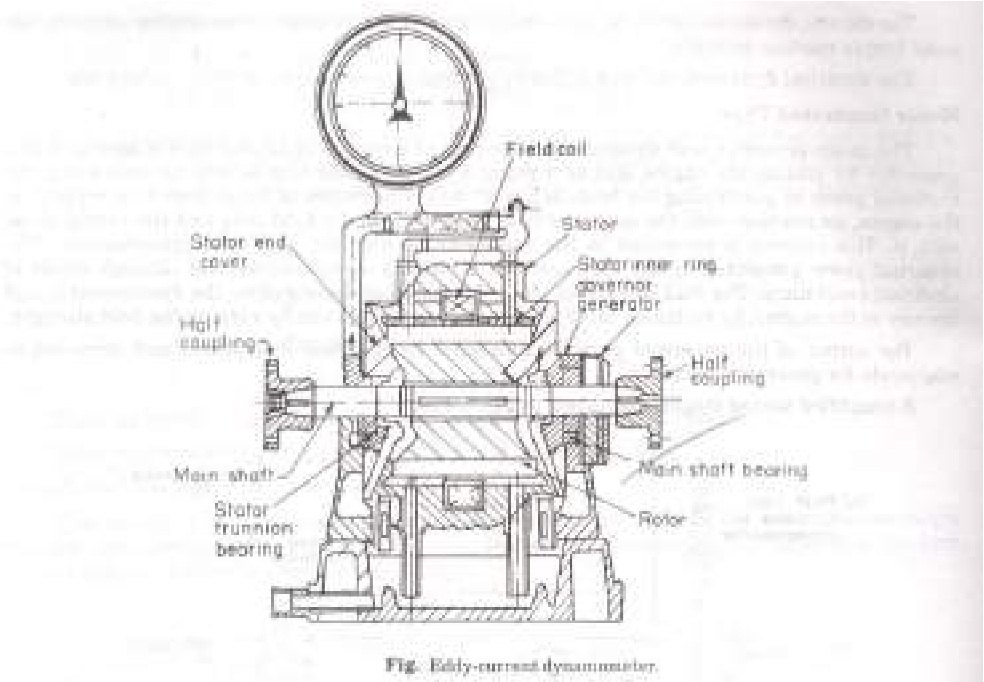


Figure.6

The power output of eddy-current dynamometer is given by the equation where *C* is eddy-current dynamometer constant.

The advantages of eddy-current dynamometer are listed below:

1. High absorbing power per unit weight of dynamometer.
2. Level of field excitation is below 1% of the total power handled by the dynamometer.
3. The torque development is smooth as eddy current developed smooth.
4. Relatively higher torque is provided under low speed conditions.
5. There is no limit to the size of dynamometer.

1.5.4 Swinging Field Dynamometer

The arrangement of swinging field dynamometer and corresponding diagram of electric connections are shown in Figure 7.

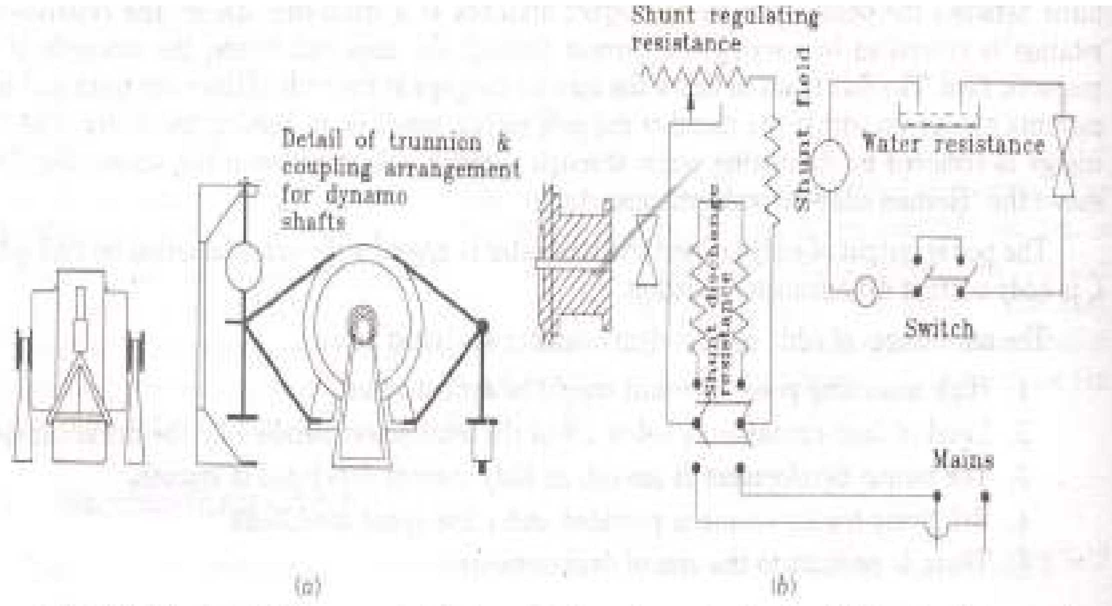


Fig. (a) Swinging-field dynamometer (b) Diagram of connections for swinging-field electrical dynamometer.

Figure.7

A swinging field DC dynamometer is basically a DC shunt motor. It is supported on trunnion bearings to measure the reaction torque that the outer casing and field coils tend to rotate with the magnetic drag. Therefore, it is named as "Swinging field". The Torque is measured with an arm and weighting equipment in the usual manner.

The choice of dynamometer depends on the use for which the machine is purchased. An electric dynamometer is preferred as it can operate as motor used for pumping or generator for testing the engine. Also, engine friction power can also be measured by operating the dynamometer in the motoring mode.

An eddy-current or hydraulic dynamometer may be used because of low initial coast and an ability to operate at high speeds. The armature of the electric dynamometer is large and heavy compared with eddy-current dynamometer and requires strong coupling between dynometer and engine.

1.6 MEASUREMENT OF I.P OF MULTI-CYLINDER ENGINE (MORSE TEST)

This method is used in multi-cylinder engines to measure I.P with out the use of indicator. The BP of the engine is measured by cutting off each cylinder in turn. If the engine consists of 4-cylinders, then the BP of the engine should be measured four times cutting each cylinder turn by turn. This is applicable to petrol as well as for diesel engines. The cylinder of a petrol engine is made inoperative by "shorting" the spark plug whereas in case of diesel engine, fuel supply is cut-off to the required cylinder.

If there are '*n*' cylinders in an engine and all are working, then

$$(B.P)_n = (I.P)_n - (F.P)_n \quad \ldots..(11)$$

Where F.P is the frictional power per cylinder.

If one cylinder is inoperative then the power developed by that cylinder (IP) is lost and the speed of the engine will fall as the load on the engine remains the same. The engine speed can be resorted to its original value by reducing the load on the engine by keeping throttle position same. This is necessary to maintain the FP constant, because it is assumed that the FP is independent of load and depends only on speed of the engine.

When cylinder "1" is cut off; then

$$(B.P)_{n-1} = (I.P)_{n-1} - (F.P)_n \quad \ldots..(12)$$

By subtracting Eq. (23.7) from Eq.(23.6), we obtain the IP of the cylinder which is not firing i.e., $(B.P)_n - (B.P)_{n-1} = (IP)_n - (IP)_{n-1} = I.P_1$
Similarly IP of all other cylinders can be measured one by one then the sum of IPs of all cylinders will be the total IP of the engine.

This method of obtaining IP of the multicylinder engine is known as 'Morse Test'.

1.8 MEASUREMENT OF FUEL CONSUMPTION

Two glass vessels of 100cc and 200cc capacity are connected in between the engine and main fuel tank through two, three-way cocks. When one is supplying the fuel to the engine, the other is being filled. The time for the consumption of 100 or 200cc fuel is measured with the help of stop watch.

A small glass tube is attached to the main fuel tank as shown in figure. When fuel rate is to be measured, the valve is closed so that fuel is consumed from the burette. The time for a known value of fuel consumption can be measured and fuel consumption rate can be calculated.

$$\text{Fuel consumption kg/hr} = \frac{X_{cc} \text{ X Sp. gravity of fuel}}{1000 \text{ x } t}$$

1.9 MEASUREMENT OF HEAT CARRIED AWAY BY COOLING WATER

The heat carried away by cooling water is generally measured by measuring the water flow rate through the cooling jacket and the rise in temperatures of the water during the flow through the engine.

The inlet and out let temperatures of the water are measured by the thermometers inserting in the pockets provided at inlet to and outlet from the engine. The quantity of water flowing is measured by collecting the water in a bucket for a specified period or directly with the help of flow meter in case of large engine. The heat carried away by cooling water is given by

Where $Q_w = C_p m_w(T_{wo} - T_{wi})$ kJ/min.
M_w = mass of water/min.
T_{wi} = Inlet temperature of water, ^oC
T_{wo} = Out let temperature of water, ^oC
C_p = Specific heat of water.

1.10 MEASUREMENT OF HEAT CARRIED AWAY BY EXHAUST GASES

The mass of air supplied per kg of fuel used can be calculated by using the equation if the exhaust analysis is made

$$m_a = \frac{N \times C}{33\,(C_1 + C_2)}$$

And heat carried away by the exhaust gas per kg of fuel supplied can be calculated as

$$Q_g = (m_a + 1)\, C_{pg}\, (T_{ge} - T_a) \text{ kJ/kg of fuel} \qquad \text{….(16)}$$

Where $(m_a + 1)$ = mass of exhaust gases formed per kg of fuel supplied to engine
C_{pg} = Specific heat of exhaust gases
T_{ge} = Temperature of exhaust gases coming out from the engine ^oC.
T_a = Ambient temperature ^oC or engine room temperature.

The temperature of the exhaust gases is measured with the help of suitable thermometer or thermocouple.

Another method used for measuring the heat carried away by exhaust gases is to measure the fuel supplied per minute and also to measure the air supplied per minute with the help of air box method. The addition of fuel and air mass will be equal to the mass of exhaust gases.

And exhaust gas calorimeter is commonly used in the laboratory for the measurement of heat carried by exhaust gases.

1.10.1Exhaust Gas Calorimeter

The exhaust gas calorimeter is a simple heat exchanger in which, part of the heat of the exhaust gases is transferred to the circulating water. This calorimeter helps to determine the mass of exhaust gases coming out of the engine.

The arrangement of the exhaust gas calorimeter is shown in fig. 23.5.

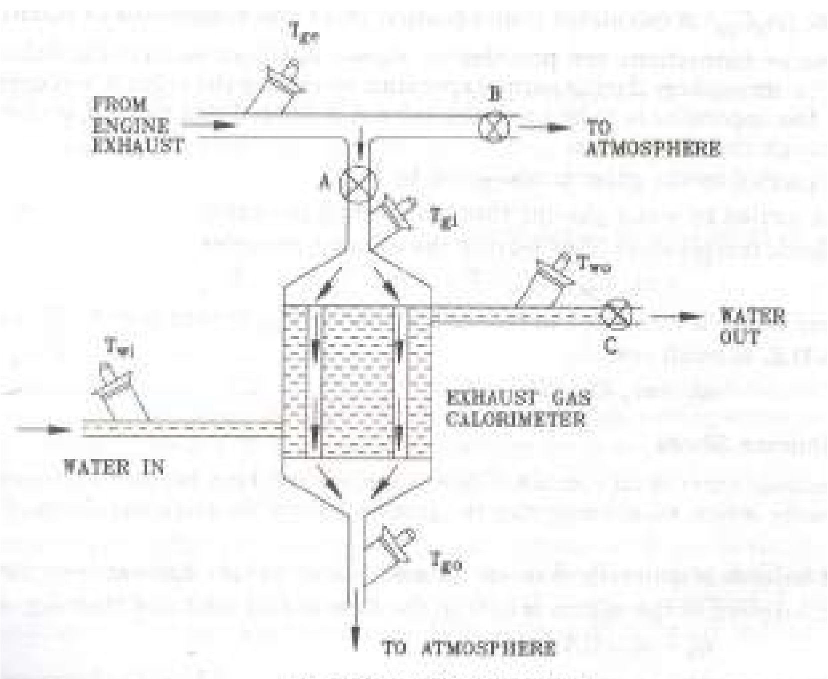


Fig. Exhaust gas calorimeter.

Figure.8

The exhaust gases from the engine exhaust are passed through the exhaust gas calorimeter by closing the valve *B* and opening the valve A. The hot gases are cooled by the water flow rate is adjusted with the help of valve of *'C'* to give a measurable temperature rise to water circulated.

If it is assumed that the calorimeter is well insulated, there is no heat loss except by heat transfer from the exhaust gases to the circulating water, then

Heat lost by exhaust gases = Heat gained by circulating water.

Therefore $m_g , C_{pg} (T_{gi} - T_{go}) = m_w . \ C_{pw} (T_{wo} - T_{wi})$

Where T_{gi} = The temperature of the exhaust gases entering the calorimeter, ^oC

T_{go} = The temperature of the exhaust gases leaving the calorimeter, ^oC

T_{wi} = The temperature of water entering the calorimeter, ^oC

T_{wo} = The temperature of water leaving the calorimeter, ^oC

m_w = Mass of water circulated through the exhaust gas calorimeter, generally measured.

m_g = Mass of exhaust gases (unknown)

C_{pg} = specific heat of exhaust gases.

C_{pw} = Specific heat of water.

$$\therefore m_g = \frac{C_{pw}}{C_{pg}} \left(\frac{T_{wo} - T_{wi}}{T_{gi} - T_{go}} \right) m_w \qquad \ldots(17)$$

As all the quantities on the RHS are known the gas flow rate can be determined.

Then the heat carried away by the exhaust gases is given by

$$Q_g = m_g \ C_{pg.} \ (T_{ge} - T_a) \qquad \ldots.(18)$$

Where T_{ge} = Temperature of exhaust gases just leaving the engine exhaust valve, ^oC

T_a = Ambient temperature, ^oC

Usually valve connections are provided as shown in figure so that the exhaust gases are exhausted to the atmosphere during normal operation by closing the valve *A* and opening the valve *B*. Only when the apparatus is to be used, the valve *A* is opened and valve *B* is closed so that the gases pass through the calorimeter.

The heat carried by the gases is also given by
Q_g = Heat carried by water passing through exhaust gas calorimeter + Heat in exhaust gases above atmoshpheric temperature after leaving the exhaust gas calorimeter.

$$= m_w C_{pw} (T_{wo} - T_{wi}) + m_g C_{pg} (T_{go} - T_a) \quad \ldots(19)$$

If sufficient water is circulated to reduce the value of T_{go} to very near to T_a, then the second term on the RHs is small and,

$$Q_g = m_w C_{pw} (T_{wo} - T_{wi}) \quad \ldots.(20)$$

1.11 HEAT BALANCE SHEET

A heat balance sheet is an account of heat supplied and heat utilized in various ways in the system. Necessary information concerning the performance of the engine is obtained from the heat balance.
The heat balance is generally done on second basis or minute basis or hour basis.
The heat supplied to the engine is only in the form of fuel-heat and that is given by

$$Q_s = m_f \text{ X CV}$$

Where mf is the mass of fuel supplied per minute or per sec. and CV is the lower calorific value of the fuel.
The various ways in which heat is used up in the system is given by

(a) Heat equivalent of BP = kW = kJ/sec. = 0 kJ/min.

(b) Heat carried away by cooling water
= C_{pw} X $m_w (T_{wo} - T_{wi})$ kJ/min.

Where mw is the mass of cooling water in kg/min or kg/sec circulated through the cooling jacket and $(T_{wo} - T_{wi})$ is the rise in temperature of the water passing through the cooling jacket of the engine and C_{pw} is the specific heat of water in kJ/kg-K.

(c) Heat carried away by exhaust gases
= $m_g C_{pg} (T_{ge} - T_a)$ (kJ/min.) or (kJ/sec)

Where mg is the mass of exhaust gases in kg/min. or kg/sec and it is calculated by using one of the methods already explained.

T_g = Temperature of burnt gases coming out of the engine.
T_a = Ambient Temperature.
C_{pg} = Sp. Heat of exhaust gases in (kJ/kg-K)

(d) A part of heat is lost by convection and radiation as well as due to the leakage of gases. Part of the power developed inside the engine is also used to run the accessories as lubricating pump, cam shaft and water circulating pump. These cannot be measured precisely and so this is known as unaccounted 'losses'. This unaccounted heat energy is calculated by the different between heat supplied Q_s and the sum of *(a)* + *(b) (c).*

The results of the above calculations are tabulated in a table and this table is known as "Heat Balance Sheet". It is generally practice to represent the heat distribution as percentage of heat supplied. This is also tabulated in the same heat balance sheet.

Heat input per minute	*kcal (kj)*	*%*	*Heat expenditure per minute*	*kcal (kj)*	*%*
Heat supplied by the combustion fuel	Q_s	100%	(a) Heat in BP.	--	--
			(b) Heat carried by jacket cooling water	--	--
			(c) Heat Carried by exhaust gases	--	--
			(d) Heat unaccounted for $= Q_s - (a + b + c)$	--	--
Total	Q_s	100%			100%

A sample tabulation which is known as a heat balance sheet for particular load condition is shown below:

NOTE: The heat in frictional FP (IP – BP) should not be included separately in heat balance sheet because the heat of FP (frictional heat) will be dissipated in the cooling water, exhaust gases and radiation and convection. Since each of these heat quantities are separately measured and heat in FP is a hidden part of these quantities; the separate inclusion would mean that it has been included twice.

The arrangement either for measuring the air or measuring the mass of exhaust gas is sufficient to find the heat carried away by exhaust gases. In some cases, both arrangements are used for cross-checking. Heat carried away by exhaust gases is calculated with the help of volumetric analysis of the exhaust gases provided the fraction of carbon in the fuel used is known.

1.12 . Indicated Specific Fuel Consumption: This is defined as the mass of fuel consumption per hour in order to produce an indicated power of one kilo watt.

Thus, indicated specific fuel consumption = isfc = $\dfrac{3600\,\dot{m}}{ip}$ kg/kWh …..(13)

1.13.Brake Specific fuel consumption:- This defined as the mass of fuel consumed per hour,
in order to develop a brake power of one kilowatt.

Thus, brake specific fuel consumption = bsfc = $\dfrac{3600\,\dot{m}}{bp}$ kg/kWh ……..(14)

1.14. Thermal Efficiency : There are two definitions of thermal efficiency as applied to IC engines. One is based on indicated power and the other on brake power.The one based on indicated power is called as *‘indicated thermal efficiency”*, and the one based on brake power is known as *“brake thermal efficiency”*.

Indicated thermal efficiency is defined as the ratio of indicated power to the energy available due to combustion of the fuel.

$$\text{Thus } \eta_{ith} = \frac{\text{Indicated Power in kW}}{\text{(Mass flow rate of fuel in kg/s) x (Calorific value of fuel in kJ/kg)}}$$

$$\text{Or} \quad \eta_{ith} = \frac{ip}{m \times CV} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

Similarly brake thermal efficiency is defined as the ratio of brake power to energy available due to combustion of the fuel.

$$\text{Or} \quad \eta_{bth} = \frac{bp}{m \times CV} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

1.15.Mechanical Efficiency: Mechanical efficiency takes into account the mechanical losses in an engine. The mechanical losses include (i) frictional losses, (ii) power absorbed by engine auxillaries like fuel pump, lubricating oil pump, water circulating pump, magneto and distributor, electric generator for battery charging, radiator fan etc., and (iii) work requited to charge the cylinder with fresh charge and work for discharging the exhaust gases during the exhaust stroke. It is defined as the ratio of brake power to indicated power. Thus

$$\eta_{mech} = \frac{bp}{ip} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

1.16. Volumetric efficiency: Volumetric efficiency is the ratio of the actual mass of air drawn into the cylinder during a given period of time to the theoretical mass which should have been drawn in during the same interval of time based on the total piston displacement, and the pressure and temperature of the surrounding atmosphere.

Thus $$\eta_v = \frac{V_{actual}}{V_{th}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

where n is the number of intake strokes per minute and V_s is the stroke volume of the piston.

2. Illustrative examples:

Example 1:- *The following observations have been made from the test of a four cylinder, two – stroke petrol engine. Diameter of the cylinder = 10 cm; stroke = 15 cm; speed = 1600 rpm; Area of indicator diagram = 5.5 cm^2; Length of the indicator diagram = 55 mm; spring constant = 3.5 bar/cm; Determine the indicated power of the engine.*

Given:- d = 0.1 m; L = 0.15 m ; No. of cylinders = K = 4; N = 1600 rpm; n = N (two – stroke); a = 5.5 cm^2; length of the diagram = l_d = 5.5. cm; spring constant = k_s = 3.5 bar/cm ;
To find: indicated power, ip.

Solution: Indicated mean effective pressure = $p_{im} = \dfrac{a\ k_s}{l_d}$

$$\text{or} \quad p_{im} = \frac{5.5 \times 3.5}{5.5} = 3.5 \text{ bar} = 3.5 \times 10^{5} \text{ N / m}^2$$

$$\text{Indicated power} = ip = \frac{p_{im}\,LAnK}{60{,}000} = \frac{3.5 \times 10^5 \times 0.15 \times (\pi/4) \times 0.1^2 \times 1600 \times 4}{60{,}000}$$

$$= 43.98 \text{ kW}$$

Example 2:- *A gasoline engine (petrol engine) working on Otto cycle consumes 8 litres of petrol per hour and develops 25 kW. The specific gravity of petrol is 0.75 and its calorific value is 44,000 kJ/kg. Determine the indicated thermal efficiency of the engine*

Given:- Volume of fuel consumed/hour = y/t = $8 \times 10^{3} / 3600$ cc/s ;

ip = 25 kW; CV = 44,000 kJ/kg;

Specific gravity of petrol = s = 0.75

To find: η_{ith} ;

Solution: Mass of fuel consumed = $\dot{m} = \dfrac{y\,s}{1000\,t} = \dfrac{8 \times 10^{3} \times 0.75}{1000 \times 3600} = 1.67 \times 10^{-3}$ kg/s.

$$\text{Indicated thermal efficiency} = \eta_{ith} = \frac{ip}{\dot{m}\ CV} = \frac{25}{1.67 \times 10^{-3} \times 44000}$$

$$= 0.3402 = 34.02 \text{ \%.}$$

Example 2.3:- *The bore and stroke of a water cooled, vertical, single-cylinder, four stroke diesel engine are 80 mm and 110 mm respectively.The torque is 23.5 N-m.Calculate the brake mean effective pressure.*
What would be the mean effective pressure and torque if the engine rating is 4 kW at 1500 rpm?

Given:- Diameter = d = $80 \times 10^{-3} = 0.008$ m ; stroke = L = 0.110 m; T = 23.5 N-m;

To find (i) bmep ; (ii) bmep if bp = 4 kw and N= 1500 rpm.

Solution: (i) Relation between brake power (bp) and brake mean effective pressure (bmep) is given by

$$bp = \frac{2\pi NT}{60{,}000} = \frac{(bmep)LAn}{60{,}000}$$

Hence $bmep = (2\pi NT) / (LAn) = (2\pi NT) / \{(L\pi d^2 /4)\ N/2\}$

$$= \frac{16T}{d^2L} = \frac{16 \text{ x } 23.5}{0.08^2 \text{ x } 0.11} = 5.34 \text{ x } 10^5 \text{ N / m}^2 = 5.34 \text{ bar}$$

(ii) when bp = 4 kw and N = 1500 rpm, we have

$$bmep = \frac{60{,}000\ bp}{LAn} = \frac{60{,}000 \text{ x } 4}{0.110 \text{ x } (\pi/4) \text{ x } 0.08^2 \text{ x } (1500 / 2)}$$

$$= 5.79 \text{ x } 10^5 \text{ N/m}^2 = 5.79 \text{ bar.}$$

Also $bp = 2\pi NT / 60{,}000$ or $T = \frac{60{,}000\ bp}{2\pi N} = \frac{60{,}000 \text{ x } 4}{2 \text{ x } \pi \text{ x } 1500} = 25.46$ N – m.

Example 4:-*Find the air fuel ratio of a four stroke, single cylinder, air cooled engine with fuel consumption time for 10 cc is 20.4 s and air consumption time for 0.1 m^3 is 16.3 s. The load is 7 N at the speed of 3000 rpm. Find also the brake specific fuel consumption in kg/kWh and brake thermal efficiency.Assume the density of air as 1.175 kg/m^3 and specific gravity of the fuel to be 0.7. The lower heating value of the fuel is 43 MJ/kg and the dynamometer constant is 5000.*

Given:- y = 10 cc ; t = 20.4 s ; $V_a = 0.1$ m^3; $t_a = 16.3$ s ; W = 7 N ; N = 3000 rpm;

$\rho_a = 1.175$ kg/m^3; s = 0.7 ; CV = 43 x 10^3 kJ/kg; Dynamometer constant = C = 5000.

To find:- (i) m_a / m_f ; (ii) bsfc ; (iii) $\eta_{bth.}$

Solution: (i) Mass of air consumed = $m_a = \frac{0.1 \text{ x } 1.175}{16.3} = 7.21 \text{ x } 10^{-3}$ kg/s.

Mass of fuel consumed = $m_f = \frac{y\ s}{1000\ t} = \frac{10 \text{ x } 0.7}{1000 \text{ x } 20.4} = 0.343 \text{ x } 10^{-3}$ kg/s

$$\text{Air fuel ratio} = \frac{m_a}{m_f} = \frac{7.21 \times 10^{-3}}{0.343 \times 10^{-3}} = 21$$

$$\text{(ii) Brake power} = bp = WN / C = \frac{7 \times 3000}{5000} = 4.2 \text{ kW}$$

$$bsfc = \frac{m_f \times 3600}{bp} = \frac{0.343 \times 10^{-3} \times 3600}{4.2} = 0.294 \text{ kg/kWh}$$

$$\text{(iii) } b_{ith} = \frac{bp}{m_f\, CV} = \frac{4.2}{0.343 \times 10^{-3} \times 43 \times 10^{3}} = 0.2848 = 28.48\ \%.$$

Example 2.5:- *A six cylinder, gasoline engine operates on the four stroke cycle. The bore of each cylinder is 80 mm and the stroke is 100 mm. The clearance volume in each cylinder is 70 cc. At a speed of 4000 rpm and the fuel consumption is 20 kg/h. The torque developed is 150 N-m. Calculate (i) the brake power, (ii) the brake mean effective pressure, (iii) brake thermal efficiency if the calorific value of the fuel is 43000 kJ/kg and (iv) the relative efficiency if the ideal cycle for the engine is Otto cycle.*

Given:- K = 6 ; n = N /2 ; d = 8 cm ; L = 10 cm ; V_c = 70 cc ; N = 4000 rpm ; m_f = 20 kg/h ; T = 150 N-m ; CV = 43000 kJ/kg ;

To find:- (i) bp ; (ii) bmep ; (iii) η_{bth} ; (iv) $\eta_{\text{Relative.}}$

Solution:

$$\text{(i) } bp = \frac{2\pi NT}{60{,}000} = \frac{2 \times \pi \times 4000 \times 150}{60{,}000}$$

$$= 62.8 \text{ kW}$$

$$\text{(ii) bmep} = \frac{60{,}000\ bp}{L\, A\, n\, K} = \frac{60{,}000 \times 62.8}{0.1 \times (\pi / 4) \times 0.08^{2} \times (4000/2) \times 6}$$

$$= 6.25 \times 10^{5} \text{ N/m}^2 = 6.25 \text{ bar}$$

$$\text{(iii) } \eta_{bth} = \frac{bp}{m_f\, CV} = \frac{62.8}{(20 / 3600) \times 43{,}000} = 0.263 = 26.3\ \%.$$

(iv) Stroke volume = $V_s = (\pi / 4)\, d^2 L = (\pi / 4) \times 8^2 \times 10 = 502.65$ cc

$$\text{Compression Ratio of the engine} = R_c = \frac{V_s + V_c}{V_c} = \frac{502.65 + 70}{70} = 8.18$$

Air standard efficiency of Otto cycle = $\eta_{Otto} = 1 - (1/ R_c^{\gamma - 1})$

$$= 1 - \frac{1}{8.18^{0.4}} = 0.568 = 56.8\ \%$$

Hence Relative efficiency = $\eta_{Relative} = \eta_{bth} / \eta_{Otto} = 0.263 / 0.568 = 0.463 = 46.3$ %.

Example 2.6:- *An eight cylinder, four stroke engine of 9 cm bore, 8 cm stroke and with a compression ratio of 7 is tested at 4500 rpm on a dynamometer which has 54 cm arm. During a 10 minute test, the dynamometer scale beam reading was 42 kg and the engine consumed 4.4 kg of gasoline having a calorific value of 44,000 kJ/kg. Air at 27 C and 1 bar was supplied to the carburetor at a rate of 6 kg/min. Find (i) the brake power, (ii) the brake mean effective pressure, (iii) the brake specific fuel consumption, (iv) the brake specific air consumption, (v) volumetric efficiency , (vi) the brake thermal efficiency and (vii) the air fuel ratio.*

Given:- $K = 8$; Four stroke hence $n = N/2$; $d = 0.09$ m; $L = 0.08$ m; $R_c = 7$; $N = 4500$ rpm; Brake arm = $R = 0.54$ m ; $t = 10$ min ; Brake load = $W = (42 \times 9.81)$ N

$m_f = 4.4$ kg ; CV = 44,000 kJ/kg ; $T_a = 27 + 273 = 300$ K ; $p_a = 1$ bar; $\dot{m}_a = 6$ kg/min;

To find:- (i) bp ; (ii) bmep ; (iii) bsfc ; (iv) bsac ; (v) η_v ; (vi) η_{bth} ; (vii) $\dot{m}_a / \dot{m}_f$

Solution:

$$\text{(i) bp} = \frac{2\pi\, NT}{60{,}000} = \frac{2\pi\, NWR}{60{,}000} = \frac{2 \times \pi \times 4500 \times (42 \times 9.81) \times 0.54}{60{,}000}$$

$$= 104.8 \text{ kW}$$

$$\text{(ii) bmep} = \frac{60{,}000\ \text{bp}}{L\,A\,n\,K} = \frac{60{,}000 \times 104.8}{0.08 \times (\pi / 4) \times 0.09^2 \times (4500 / 2) \times 8}$$

$$= 6.87 \times 10^5 \text{ N/m}^2 = 6.87 \text{ bar.}$$

(iii) mass of fuel consumed per unit time = $\dot{m}_f = m_f / t = 4.4 \times 60 / 10$ kg/h

$= 26.4$ kg/h

Brake specific fuel consumption = bsfc = $\dfrac{\dot{m}_f}{bp} = \dfrac{26.4}{104.8} = 0.252$ kg / kWh

(iv) brake specific air consumption = bsac = $\dfrac{\dot{m}_a}{bp} = \dfrac{6 \times 60}{104.8}$

$= 3.435$ kg / kWh

(v) $\eta_{bth} = \dfrac{bp}{\dot{m}_f\ CV} = \dfrac{104.8}{(26.4 / 3600) \times 44{,}000} = 0.325 = 32.5\ \%$.

(vi) Stroke volume per unit time = $\dot{V}_s = (\pi d^2/4)\ L\ n\ K$

$$= \frac{\pi}{4} \times (0.09^2) \times 0.08 \times (4500 / 2) \times 8$$

$= 9.16\ m^3$ / min.

Volume flow rate of air per minute = $\dot{V}_a = \dfrac{\dot{m}_a R_a T_a}{p_a} = \dfrac{6 \times 286 \times 300}{1 \times 10^5}$

$= 5.17\ m^3$ / min

Volumetric efficiency = $\eta_v = \dot{V}_a / \dot{V}_s = 5.17 / 9.16 = 0.5644 = 56.44\ \%$.

(vii) Air fuel ratio = $\dot{m}_a / \dot{m}_f = 6 / (4.4 / 10) = 13.64$

Example 2.7:- *A gasoline engine working on four- stroke develops a brake power of 20.9 kW. A Morse test was conducted on this engine and the brake power (kW) obtained when each cylinder was made inoperative by short circuiting the spark plug are 14.9, 14.3, 14.8 and 14.5 respectively. The test was conducted at constant speed. Find the indicated power, mechanical efficiency and brake mean effective pressure when all the cylinders are firing. The bore of the engine is 75mm and the stroke is 90 mm. The engine is running at 3000 rpm.*

Given:- brake power when all cylinders are working = B_t = 20.9 kW ;

Brake power when cylinder 1 is inoperative = B_1 = 14.9 kW ;

Brake power when cylinder 2 is inoperative = B_2 = 14.3 kW ;

Brake power when cylinder 3 is inoperative = B_3 = 14.8 kW ;

Brake power when cylinder 4 is inoperative = B_4 = 14.5 kW ;

N = 3000 rpm ; d = 0.075 m ; L = 0.09 m ;

To find:- (i) $(ip)_{total}$; (ii) η_{mech} ; (iii) bmep ;

Solution:

(i) $(ip)_{total} = ip_1 + ip_2 + ip_3 + ip_4 = (B_t - B_1) + (B_t - B_2) + (B_t - B_3) + (B_t - B_4)$

$= 4B_t - (B_1 + B_2 + B_3 + B_4) = 4 \text{ x } 20.9 - (14.9 + 14.3 + 14.8 + 14.5)$

= 25.1 Kw

$$\text{(ii) } \eta_{mech} = \frac{B_t}{(ip)_{total}} = \frac{20.9}{25.1} = 0.833 = 83.3\ \%$$

$$\text{(iii) bmep} = \frac{60{,}000\ B_t}{L\ A\ n\ K} = \frac{60{,}000 \text{ x } 20.9}{0.09 \text{ x } (\pi / 4) \text{ x } 0.075^{2} \text{ x } (3000 / 2) \text{ x } 4}$$

$= 5.25 \text{ x } 10^{5}\ N / m^2 = 5.25$ bar.

Example 2.8:- *The following observations were recorded during a trail of a four – stroke, single cylinder oil engine.*
Duration of trial = 30 min ; oil consumed = 4 litres ; calorific value of oil = 43 MJ/kg ; specific gravity of fuel = 0.8 ; average area of the indicator diagram = 8.5 cm^2; length of the indicator diagram = 8.5 cm; Indicator spring constant = 5.5 bar/cm; brake load = 150 kg; spring balance reading = 20 kg; effective brake wheel diameter = 1.5 m ; speed = 200 rpm ; cylinder diameter = 30 cm ; stroke = 45 cm ; jacket cooling water = 10 kg/min ; temperature rise of cooling water = 36 C. Calculate (i) indicated power, (ii) brake power, (iii) mechanical efficiency, (iv) brake specific fuel consumption, (v) indicated thermal efficiency, and (vi) heat carried away by cooling water.

Given:- $t = 30$ min ; $y = 4000$ cc; $CV = 43 \times 10^{3}$ kJ/kg; $s = 0.8$; area of the diagram = a = 8.5 cm^2; length of the diagram = $l_d = 8.5$ cm ; indicator spring constant = $k_s = 5.5$ bar / cm; $W = 150 \times 9.81$ N ; Brake radius = $R = 1.5 / 2 = 0.75$ m; $N = 200$ rpm ; $d = 0.3$ m ; $L = 0.45$ m ; $\dot{m}_w = 10$ kg/min ; $\Delta T_w = 36$ C; Spring Balance Reading = $S = 20 \times 9.81$ N

To find:- (i) ip ; (ii) bp ; (iii) η_{mech} ; (iv) bsfc ; (v) η_{ith} ; (vi) $\dot{Q}_w$

Solution:

(i) $$p_{im} = \frac{a}{l_d} k_s = \frac{8.5}{8.5} \times 5.5 = 5.5 \text{ bar} = 5.5 \times 10^5 \text{ N/m}^2$$

$$ip = \frac{p_{im} L A n K}{60{,}000} = \frac{5.5 \times 10^{5} \times 0.45 \times (\pi / 4) \times 0.3^2 \times (200 / 2) \times 1}{60{,}000}$$

$$= 29.16 \text{ kW}$$

(ii) $$bp = \frac{2\pi N(W - S) R}{60{,}000} = \frac{2 \times \pi \times 200 \times (150 - 20) \times 9.81 \times 0.75}{60{,}000}$$

$$= 20.03 \text{ kW}$$

(iii) $\eta_{mech} = bp / ip = 20.03 / 29.16 = 0.687 = 68.7\ \%$.

(iv) Mass of fuel consumed per hour = $$\dot{m}_f = \frac{y\, s}{1000\, t} \times 60 = \frac{4000 \times 0.8}{1000 \times 30} \times 60$$

$$= 6.4 \text{ kg / h.}$$

$$bsfc = \dot{m}_f / bp = \frac{6.4}{20.03} = 0.3195 \text{ kg/kWh}$$

(v) $$\eta_{ith} = \frac{ip}{\dot{m}_f\, CV} = \frac{29.16}{(6.4 / 3600) \times 43 \times 10^{3}} = 0.3814 = 38.14\ \%.$$

(vi) $\dot{Q}_w = \dot{m}\, C_p\, \Delta T_w = (10 / 60) \times 4.2 \times 36 = 25.2$ kW

Example 2.9:- *A four stroke gas engine has a cylinder diameter of 25 cm and stroke 45 cm. The effective diameter of the brake is 1.6 m.The observations made in a test of the engine were as follows.*
Duration of test = 40 min; Total number of revolutions = 8080 ; Total number of explosions = 3230; Net load on the brake = 80 kg ; mean effective pressure = 5.8 bar; Volume of gas used = 7.5 m^3; Pressure of gas indicated in meter = 136 mm of water (gauge); Atmospheric temperature = 17 C; Calorific value of gas = 19 MJ/ m^3 at NTP; Temperature rise of cooling water = 45 C; Cooling water supplied = 180 kg.
Draw up a heat balance sheet and find the indicated thermal efficiency and brake thermal efficiency. Assume atmospheric pressure to be 760 mm of mercury.

Given:- $d = 0.25$ m ; $L = 0.45$ m; $R = 1.6 / 2 = 0.8$ m; $t = 40$ min ; $N_{total} = 8080$;

Hence $N = 8080 / 40 = 202$ rpm $n_{total} = 3230$;

Hence $n = 3230 / 40 = 80.75$ explosions / min; $W = 80 \times 9.81$ N; $p_{im} = 5.8$ bar ;

$V_{total} = 7.5$ m^3; hence $\dot{V} = 7.5 / 40 = 0.1875$ m^3/min; $p_{gauge} = 136$ mm of water (gauge);

$T_{atm} = 17 + 273 = 290$ K; $(CV)_{NTP} = 19 \times 10^3$ kJ/ m^3 ; $\Delta T_w = 45$ C;

$\dot{m}_w = 180 / 40 = 4.5$ kg/min; $p_{atm} = 760$ mm of mercury

To find:- (i) η_{ith} ; (ii) η_{bth} ; (iii) heat balance sheet

Solution:

(i)
$$ip = \frac{p_{im}\ L\ A\ n\ K}{60{,}000} = \frac{5.8 \times 10^5 \times (\pi / 4) \times 0.25^2 \times 0.45 \times 80.75}{60{,}000}$$

$$= 17.25 \text{ kW.}$$

$$bp = \frac{2\pi\ N\ W\ R}{60{,}000} = \frac{2 \times \pi \times 202 \times (80 \times 9.81) \times 0.8}{60{,}000}$$

$$= 13.28 \text{ kW}$$

Pressure of gas supplied = $p = p_{atm} + p_{gauge} = 760 + 136 / 13.6 = 770$ mm of mercury

Volume of gas supplied as measured at NTP = $\dot{V}_{NTP} = \dot{V}\ (T_{NTP} / T)(p / p_{NTP})$

$$= \frac{0.1875 \times 273 \times 770}{290 \times 760} = 0.17875 \text{ m}^3 / \text{min}$$

Heat supplied by fuel = $\dot{Q}_f = \dot{V}_{NTP}\ (CV)_{NTP} = 0.17875 \times 19 \times 10^3 = 3396.25$ kJ/min

Heat equivalent of bp in kJ/min = 13.28 x 60 = 796.4 kJ/min

Heat lost to cooling water in kJ/min = $\dot{m}_w C_p \Delta T_w = 4.5 \times 4.2 \times 45 = 846.5$ kJ/min

Friction power = ip – bp = 17.25 – 13.28 = 3.97 kW

Hence heat loss due to fiction, pumping etc. = 3.97 x 60 = 238.2 kJ/min

Heat lost in exhaust, radiation etc (by difference) = 3396.25 – (896.4 + 796.4 + 238.2)

= 1465.15 kJ/min

Heat Balance Sheet:

Item No.		*Heat Energy Input*		*Heat Energy spent*	
		(kJ/min)	*(percent)*	*(kJ/min)*	*(percent)*
1	Heat supplied by fuel	3396.25	100.00		
2	Heat equivalent of bp			896.4	26.4
3	Heat lost to cooling Water			796.4	23.4
4	Heat equivalent of fp			238.2	7.0
5	Heat unaccounted (by difference)			1465.15	43.2
	Total	3396.25	100.0	3396.25	100.0

Example 2.10:- *A test on a two-stroke engine gave the following results at full load.*

Speed = 350 rpm; Net brake load = 65 kg ; mean effective pressure = 3 bar ; Fuel consumption = 4 kg/h ; Jacket cooling water flow rate = 500 kg/h ; jacket water temperature at inlet = 20 C ; jacket water temperature at outlet = 40 C ; Test room temperature = 20 C ; Temperature of exhaust gases = 400 C; Air used per kg of fuel = 32 kg ; cylinder diameter = 22 cm ;stroke = 28 cm; effective brake diameter = 1 m ; Calorific value of fuel = 43 MJ/kg ; Mean specific heat of exhaust gases = 1 kJ/kg –K. Find indicated power, brake power and draw up a heat balance for the test in kW and in percentage.

Given:- Two stroke engine. Hence n = N ; N = 350 rpm ; W = (65 x 9.81) N ;

p_{im} = 3 bar ; $\dot{m}_f$ = 4 kg/h ; $\dot{m}_w$ = 500 kg/h ; T_{wi} = 20 C ; T_{wo} = 40 C ; T_{atm} = 20 C ;

T_{eg} = 400 C ; $\dot{m}_a / \dot{m}_f$ = 32 ; d = 0.22 m ; L = 0.28 m ; Brake radius = R = ½ m ;

CV = 43,000 kJ/kg ; $(C_p)_{eg}$ = 1.0 kJ/(kg-K) ;

To find:- (i) ip ; (ii) bp ; and (iii) heat balance;

Solution:

$$\text{(i) ip} = \frac{p_{im}\, LAn}{60{,}000} = \frac{3 \times 10^5 \times 0.28 \times (\pi/4) \times 0.22^2 \times 350}{60{,}000}$$

$$= 18.63 \text{ kW.}$$

$$\text{(ii) bp} = \frac{2\pi\, N\, WR}{60{,}000} = \frac{2 \times \pi \times 350 \times (65 \times 9.81) \times 0.5}{60{,}000}$$

$$= 11.68 \text{ kW.}$$

$$\text{(iii) Heat supplied in kW} = \dot{m}_f\, CV = (4 / 3600) \times 43{,}000$$

$$= 47.8 \text{ kW}$$

$$\text{Heat lost to cooling water} = \dot{m}_w\, (C_p)_w\, [T_{wo} - T_{wi}]$$

$$= (500 / 3600) \times 4.2 \times [40 - 20]$$

$$= 11.7 \text{ kW.}$$

$$\text{Heat lost in exhaust gases} = (\dot{m}_a + \dot{m}_f)\, (C_p)_{eg}\, [T_{eg} - T_{atm}]$$

$$= \frac{(32 + 1) \times 4}{3600} \times 1.0 \times [400 - 20]$$

$$= 13.9 \text{ kW}$$

Heat balance sheet:

Heat Input	kW	%	Heat Expenditure	kW	%
Heat supplied by fuel	47.8	100	Heat in bp	11.68	24.4
			Heat lost to cooling Water	11.70	24.5
			Heat lost to exhaust Gases	13.90	29.1
			Unaccounted heat (by difference)	10.52	22.0
Total	**47.80**	**100**	**Total**	**47.80**	**100.0**

Refrigeration

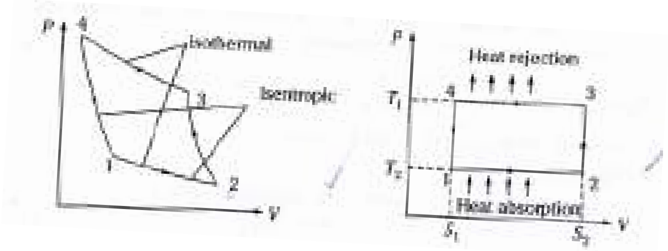
Definition

Refrigeration is the process of providing and maintaining temperature of the system below that of the surrounding atmosphere.

Carnot Cycle

The reversed carnot cycle can be considered in refrigeration system.

$$C.O.P = \frac{T_2}{T_2 - T_1} \quad \text{where}\, T_2 < T_1$$



Unit of Refrigeration

The common unit used in the field of refrigeration is known as Ton of refrigeration.

A ton of refrigeration is defined as the quantity of heat required to be removed to produce one ton (1000kg) of ice within 24 hours when the initial condition of water is 0ºC

$$Ton \text{ of refrigeration} = \frac{1000x335}{24x3600} = 3.5 \text{ kJ/s}$$

Consider a refrigerator of T tons capacity,
Refrigeration capacity = 3.5 kJ/s
Heat removed from
refrigerator = Refrigeration effect =R.E. kJ/s
Power of the compressor =work/kg of refrigerant x mass flow rate

Air Refrigeration system working on Bell-coleman cycle

In air refrigeration system, air is used as the refrigerant which always remains in the gaseous phase. The heat removed consists only of sensible heat and as a result, the coefficient of performance (C.O.P) is low.

The various processes are:

Process 1-2:

The air leaving the evaporator enters a compressor. Where it is compressed isentropically to higher pressure and temperature.

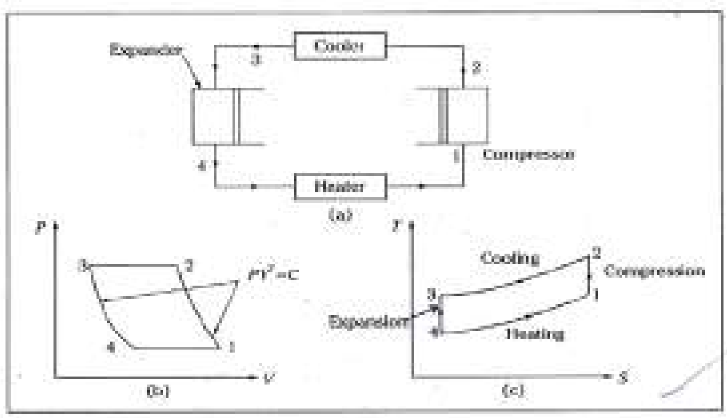
Process 2-3:

This high pressure, high temperature air, then enters a cooler where it is cooled at constant pressure to a low temperature.

Process 3-4: This high pressure, low temperature air is then expanded in an expander to lower pressure and temperature in a isentropic manner.At point 4, the temperature of the air will be lowest.

Process 4-1: This low temperature air is then passed through the heater coils where it absorbs heat from the space to be cooled namely the refrigerator and the air gets heated back to the initial temperature, but in the process, it cools the refrigerator. And the cycle repeats.

Air refrigeration system



Expression C.O.P when compression and expansion are Isentropic

Refrigeration Effect = Heat removed from the refrigerator

$$= C_p(T_1 - T_4)kJ/kg$$

$$Work\ \text{input} = \mathrm{W_C} - W_E = \gamma\left(\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right) - \gamma\left(\frac{P_3V_3 - P_4V_4}{\gamma - 1}\right)$$

$$Work\ \text{input} = \mathrm{W_C} - W_E = \left(\frac{\gamma}{\gamma - 1}\right)\left[R(T_2 - T_1) - R(T_3 - T_4)\right]$$

$$W_{net} = \left(\frac{\gamma R}{\gamma - 1}\right)\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

$$But \qquad \mathrm{C_p} = \frac{\gamma R}{\gamma - 1}$$

$$W_{net} = \mathrm{C_p}\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

Process 1 - 2 is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad ----(2)$$

Process 3 - 4 is isentropic

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad ----(3)$$

From (2) and (3)

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$C.O.P = \frac{RE}{Work} = \frac{C_p\left(T_1 - T_4\right)}{C_p\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]}$$

$$C.O.P = \frac{\left(T_1 - T_4\right)}{\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]} = \frac{1}{\frac{T_2 - T_3}{T_1 - T_4} - 1} \quad --(1)$$

$$\frac{T_2}{T_3} = \frac{T_1}{T_4}$$

$$\frac{T_2}{T_3} - 1 = \frac{T_1}{T_4} - 1$$

$$\frac{T_2 - T_3}{T_3} = \frac{T_1 - T_4}{T_4}$$

$$\frac{T_2 - T_3}{T_1 - T_4} = \frac{T_3}{T_4} \qquad -----(4)$$

From (1) and (4)

$$C.O.P = \frac{1}{\frac{T_3}{T_4} - 1}$$

$$C.O.P = \frac{T_4}{T_3 - T_4}$$

For Polytropic process

Net work

$$W_{net} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)C_p\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

$$COP = \frac{T_4}{(T_3 - T_4)\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)}$$

Advantages of air refrigeration system

1. Air is cheap, easily available.
2. It is not flammable.
3. For a given capacity, weight of air refrigeration system is less compared to other system and hence it is widely used for aircraft cooling.

Disadvantages

1. Since heat removed by air consists only of sensible heat, weight of air required is high.
2. C.O.P of the system is low compared to other systems.

Problem 1

A cold storage is to be maintained at -5°C (268k) while the surroundings are at 35°C. the heat leakage from the surroundings into the cold storage is estimated to be 29kW. The actual C.O.P of the refrigeration plant is one third of an ideal plant working between the same temperatures. Find the power required to drive the plant. (VTU Jan 2007)

Solution :-

$T_1 = 35°C = 308k \qquad T_2 = 5°C = 268k$

C.O.P of the ideal plant is nothing but

C.O.P based on carnot cycle.

$$\therefore \text{C.O.P ideal} = \frac{T_2}{T_1 - T_2}$$

$$= \frac{268}{308 - 268} = 6.7$$

$$Actual\ C.O.P = \frac{1}{3} ideal C.O.P$$

$$= \frac{1}{3} x 6.7 = 2.233$$

Q2 = The heat removed from low temperature reservoir (cold storage) must be equal to heat leakage from surroundings to the cold storage(which is 29kw)

$$Q_2 = 29kW$$

$$Actual\ C.O.P = \frac{Q_2}{W}$$

$$\mathrm{W} = \frac{Q_2}{Actual\ C.O.P} = \frac{29}{2.233}$$

$$Power\ \text{required} = 12.98\ \text{kW}$$

Problem 2

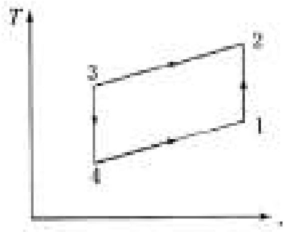
A refrigeration machine of 6 tones capacity working on Bell coleman cycle has an upper limit pressure of 5.2 bar. The pressure and temperature at the start of the compression are I bar and 18°C respectively. The cooled compressed air enters the expander at 41°C. assuming both expansion and compression to be adiabatic with an index of 1.4. Calculate:-

- (i) Co-efficient of performance.
- (ii) Quantity of air circulated per minute.
- (iii) Piston displacement of compressor and expander
- (iv) Bore of compression and expansion cylinder when the unit runs at 240 rpm and is double acting with stroke length =200 mm
- (v) Power required to drive the unit

Solution :-

$\mathrm{T}_1 = 18°C \quad \mathrm{P}_1 = 1\mathrm{bar}$

$\mathrm{T}_3 = 41°C \ \mathrm{P}_2 = 5.2\mathrm{bar}$



$$\mathit{Work}\ \text{input} = \text{C}_\text{p}\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]$$

$$= 1.005\left[\left(466 - 291\right) - \left(314 - 196\right)\right] = 57kJ\ /\ kg$$

$$C.O.P = \frac{\text{Re}\ \mathit{griferation\ effect}}{\text{Work input}}$$

$$= \frac{95.42}{57} = 1.67$$

$$\text{Re}\ \mathit{frigeration}\ \text{capacity} = 6\ \text{tons} = 6\text{x}3.5 = 21\text{kJ/s}$$

$$\mathit{Mass}\ \text{of air/sec} = \frac{\text{Re}\ \mathit{griferation}\ \mathit{c}\text{apacity}}{\text{R.E}}$$

$$= \frac{21}{95.42} = 0.22kg\ /\ s$$

$$\mathit{Power}\ \text{required} = \mathit{workdone}\text{/kg of air x Mass of air/sec}$$

$$= 57\ \text{x}\ 0.22 = 12.54\text{kW}$$

$$\text{Mass of air/min} = 0.22\text{x}60 = 13.2\text{kg/min}$$

$$V_1 = \frac{mRT_1}{P_1} = \frac{13.2x0.287x291}{1x10^2} = 11m^3\ /\ \text{min}$$

$$\mathit{Piston}\ \text{displacement of compressor V}_1 = 11m^3\ /\ \text{min}$$

$$V_4 = \frac{mRT_4}{P_4} = \frac{13.2x0.287x196}{1x10^2} = 7.42m^3\ /\ \text{min}$$

$$\mathit{Piston}\ \text{displacement of expander V}_4 = 7.42m^3\ /\ \text{min}$$

$$\mathit{But} \quad V_1 = 2\frac{\pi}{4}d_1^2 LN$$

$$11 = 2\frac{\pi}{4}d_1^2 x0.2x240$$

$$d_1 = \mathit{diameter}\ \text{of compressor cylinder} = 0.38\text{m} = 38\text{cm}$$

$$V_4 = 2\frac{\pi}{4}d_2^2 LN$$

$$7.42 = 2\frac{\pi}{4}d_1^2 x0.2x240$$

$$d_1 = \mathit{diameter}\ \text{of expander cylinder} = 0.313\text{m} = 31.3\text{cm}$$

Problem3 An air refrigerator system operating on Bell Coleman cycle, takes in air from cold room at 268 K and compresses it from 1 bar to 5.5 bar the index of compression being 1.25. the compressed air is cooled to 300 K. the ambient temperature is 20ºC. Air expands in expander where the index of expansion is 1.35.

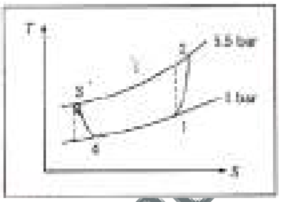
Calculate:

- ***i) C.O.P of the system***
- ***ii) Quantity of air circulated per minute for production of 1500 kg of ice per day at 0°C from water at 20ºC.***
- ***iii) Capacity of the plant.***

Solution

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 268(5.5)^{\frac{1.25-1}{1.25}}$$

$$= 376.8\text{K}$$



$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = 300\left(\frac{1}{5.5}\right)^{\frac{1.35-1}{1.35}} = 192.83\text{K}$$

$$\text{W}_\text{C} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)C_p(T_2 - T_1)$$

$$= \left(\frac{1.25}{1.25-1}\right)\left(\frac{1.4-1}{1.4}\right)1.005(376.8 - 268) = 156.2kJ/kg$$

$$\text{W}_\text{E} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)C_p(T_3 - T_4)$$

$$= \left(\frac{1.35}{1.35-1}\right)\left(\frac{1.4-1}{1.4}\right)1.005(300 - 192.83) = 118.69kJ/kg$$

$$Network = W_C - W_E = 156.2 - 118.69 = 37.5kJ/kg$$

$$R.E = C_p(T_1 - T_4) = 1.005(268 - 192.83) = 75.54kJ/s$$

$$C.O.P = \frac{RE}{work} = \frac{75.54}{37.5} = 2$$

$\textit{Heat}\ \text{extracted/kg of ice} = C_{pw}(20-0) + L$

$$= 4.187(20) + 335 = 418.74\text{kJ/kg}$$

$$\text{Mass of ice produced/sec} = \frac{1500}{24\text{x}3600} = 0.0173kg/s$$

$\textit{Actual heat}\ \text{extracted/sec} = 418.74\text{x}0.0173$

$$\text{or Refrigeration capacity} = 7.26\text{kJ/s} = \frac{7.26}{3.5} = 2.02\,tons$$

$$\textit{Mass}\ \text{flow rate} = \frac{\text{Refrigeration Capacity}}{\text{Refrigeration efect}} = \frac{7.26}{75.54}$$

$$= 0.096kg/s$$

Problem 4

An air refrigeration system is to be designed according to the following specifications
Pressure of air at compressor inlet=101kPa
Pressure of work at compressor outlet=404kPa
Pressure loss in the inter cooler=12kPa
Pressure loss in the cold chamber=3kPa
Temperature of air at compressor inlet=7°
Temperature of air at turbine inlet=27°

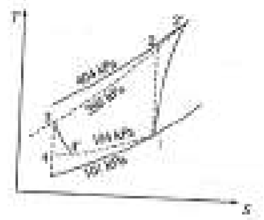
Isentropic efficiency of compressor =85%
Isentropic efficiency of turbine =85%
Determine

i) C.O.P of cycle
ii) Power required to produce 1 ton of refrigeration
iii) Mass flow rate of air required for 1 ton of refrigeration

Solution :-

$T_1 = -7°C \quad P_1 = 101\text{kPa}$

$T_3 = 27°C\ \eta_T = 0.85; \eta_C = 0.85$



$$\text{Process 1-2 is isentropic, Hence} T'_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 266\left(\frac{404}{101}\right)^{\frac{1.4-1}{1.4}} = 395.4\text{K}$$

$$\eta_C = \frac{T_2 - T_1}{T'_2 - T_1} \quad \text{or} \, T'_2 - T_1 = \frac{395.4 - 266}{0.88}$$

$$T'_2 = 418.2k$$

$$P_4 - P_1 = 0.03P_1 \quad \therefore P_4 = 1.03P_1 = 1.03x101 = 104kPa$$

$$P_2 - P_3 = 0.03P_2 \quad \therefore P_3 = 0.97P_2 = 0.97x404 = 392kPa$$

$$\text{Pr}\,ocess\ \text{3 - 4 is isentropic,} \quad \therefore T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300\left(\frac{104}{392}\right)^{\frac{1.4-1}{1.4}} = 202.3\text{K}$$

$$\eta_E = \frac{T_3 - T'_4}{T_3 - T_4} \quad \therefore T'_4 = T_3 - \eta_T\left(T_3 - T_4\right)$$

$$T'_4 = 300 - 0.85x[300 - 205.3] = 216.53k$$

$$\text{Re}\ frigeration\ \text{effect/kg of air} = \text{C}_\text{p}\left(T_1 - T_4\right)$$

$$= 1.005\text{x}[266 - 216.53] = 50.47\text{kJ/kg}$$

$$C\text{ompressor work/kg of air} = \text{C}_\text{p}\left(T'_2 - T_1\right)$$

$$= 1.005\text{x}[418.2 - 266] = 152.96\text{kJ/kg}$$

$$T\text{urbine work/kg of air} \quad \text{W}_\text{T} = \text{C}_\text{p}\left(T_3 - T_4'\right)$$

$$= 1.005\text{x}[300 - 216.53] = 84.9\text{kJ/kg}$$

$$\text{Net work Input/kg of air W}_\text{net} = \text{W}_\text{C} - \text{W}_\text{T}$$

$$= 152.96 - 80.9 = 72.06kJ/kg$$

$$C.O.P = \frac{RE}{Work} = \frac{46.73}{72.06} = 0.73$$

$$Power\ \text{required per tons of refergeration}$$

$$= \frac{\text{Refrigeration capacity}}{\text{C.O.P}}$$

$$\text{Refrigeration capacity} = 1\ \text{ton} = 3.5\text{kJ/s}$$

$$\text{Mass of air} = \frac{\text{Refrigeration capacity}}{\text{RE}}$$

$$= \frac{3.5}{50.47} = 0.075kg/s$$

$$Power = W_{net}xmassofair/\sec = 72.06x0.075 = 5.42kW$$

Problem 1

Moist air at 30°C,1.01325 bar has a relative humidity of 80%. Determine without using the psychrometry chart

1) Partial pressures of water vapour and air
2) Specific humidity
3) Specific Volume and
4) Dew point temperature (V.T.U. July2004)

Solution: At 30°C from table $p_{\upsilon s} = 4.2461 kPa$

$$\phi = \frac{p_\upsilon}{p_{\upsilon s}}$$

$$p_\upsilon = 0.8 x 4.2461 = 3.397 kPa$$

$$\omega = \frac{0.622 p_\upsilon}{p - p_\upsilon} = 0.622 x \frac{3.397}{101.325 - 3.397}$$

$$= 0.213 \text{ kg/kg of dry air.}$$

Corresponding to Pv =3.397 kPa from tables, we get dew point temperature = 28.9°C

Problem 2:

Atmospheric air at 101.325 kPa ha 30°C DBT and 15°C DPT. Without using the pschrometric chart, using the property values from the table, Calculate

1. Partial pressure of air and water vapour
2. Specific humidity
3. Relative humidity
4. Vapour density and
5. Enthalpy of moist air

Solution:

$p = 101.325 kpa = 1.01325 bar$

$DBT = 30°C,$

$\text{DPT} = 15°\text{C}$

From table

Corresponding to DBT = 30°C, we have $p_{\upsilon s} = 0.042461 bar$

Corresponding to DPT = 15°C, we have $p_\upsilon = 0.017051 bar$

Partial pressure of air $= p - p_\upsilon = 1.01325 - 0.017051$

$= 0.984274 \text{ bar}$

$$Specific\ humidity = 0.622\frac{p_\upsilon}{p_a} = \frac{0.622x0.017051}{0.984274}$$

$$= 0.01077\text{kJ/kg of dry air}$$

$$\text{Re}\,lative\ humidity = \frac{p_\upsilon}{p_{\upsilon s}} = \frac{0.017051}{0.042461} = 0.4015$$

$$= 40.15\%$$

$$\text{Enthalphy} = 1.005\text{t}_{\text{db}} + \omega(2500 + 1088t_{db})$$

$$= 1.005\text{x}30 + 0.010775(2500 + 1.88\text{x}30)$$

$$= 57.69\text{kj/kg of dry air}$$

$$Specific\ \text{volume of dry air}, \upsilon_{\text{a}} = \frac{RT}{P}$$

$$= \frac{0.2872\text{x}303}{0.98425x100} = 0.874m^3/kg$$

$$Vapour\ \text{density}\ \rho_{\text{w}} = \frac{\omega}{\upsilon_a} = \frac{0.010775}{0.847} = 0.12kg/m^3$$

Problem 3:

Air at 30°C DBT and 25°C WBT is heated to 40°C. if the air is 300 m3/min, find the amount of heat added/min and RH and WBT of air. Take air pressure to be 1 bar

Solution:

At 25°C WBT from tables pwswbt=0.03166 bar

$$\therefore\ \text{p}_\upsilon = (P_{VS})_{wbt} - \frac{(p - p_{\upsilon swbt})(t_{db} - t_{wb})}{1547 - 1.44t_{wb}}$$

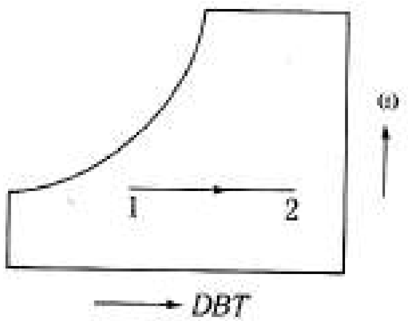
$$= 0.03166 - \frac{(1 - 0.03166)(30 - 25)}{1547 - 1.44x25}$$

$$= 0.0284\ \text{bar}$$

$$\omega_1 = 0.622\frac{p_\upsilon}{p - p_\upsilon}$$

$$= 0.622\left(\frac{0.0284}{1 - 0.0284}\right)$$

$$= 0.0179kJ/kg\ \text{of dry air}$$



$At\ 40°C\ DBT$

$P_{VS} = 0.07375 bar$

$During\ sensible\ heating\ \omega$ and p_υ $remain$ constant

$p_\upsilon = 0.0284 bar$

$$RH = \phi = \frac{p_\upsilon}{p_{\upsilon s}} = \frac{0.0284}{0.07375}$$

$$= 0.385 = 38.5\%$$

$$\mathrm{H}_2 = 1.005x40 + 0.0179(2500 + 1.88x40)$$

$$= 86.29 \text{kJ/kg of dry air}$$

$$Weight \text{ of } 300\text{m}^3/\min\ of air = \frac{(p - p_\upsilon)V}{RT}$$

$$= \frac{(1 - 0.0284)\text{x}300\text{x}10^2}{0.287x303} = 335.18 kg/\min$$

$$\therefore Heat \text{ added/min} = 335.18(86.29 - 76) = 3449 \text{kJ/min}$$

From chart WBT = 27.2°C

Problem 4:

One stream of air at 5.5m3/min at 15°C and 60% RH flows into another stream of air at 35m3/min at 25°C and 70%RH, calculate for the mixture

1) Dry bulb temperature, 2) Wet bulb temperature 3) Specific Humidity and 4) Enthalpy

Solution:

For air at 15°C and 60%RH, V=5.5m3/min

$$\therefore\ \mathrm{p}_{\upsilon s} = 0.017051 bar$$

$$RH = \phi = \frac{p_\upsilon}{p_{\upsilon s}}$$

$$\therefore\ p_\upsilon = 0.6x0.017051 = 0.01023 bar$$

$$Mass \text{ of air} = \frac{(\mathrm{p} - \mathrm{p}_\upsilon)V}{RT} = \frac{(1.01325 - 0.01023)x10^2x5.5}{0.287x288}$$

$$m_1 = 6.672 kg/\min$$

$$\omega_1 = \frac{0.622 p_\upsilon}{(\mathrm{p} - \mathrm{p}_\upsilon)} = \frac{0.622x0.01023}{(1.01325 - 0.01023)}$$

$$= 0.006343 kg/kg \text{ of dry air}$$

$$H_1 = 1.005t_{db} + \omega_1(2500 + 1.88t_{db})$$
$$= 1.008\text{x}18 + 0.006343(2500 + 1.88\text{x}15)$$
$$= 34.12\text{J/kg of dry air}$$

For air at 25°C and 70% RH, $\text{V} = 35\text{m}^3 / \text{min}$

$$P_{\upsilon s} = 0.03169bar$$

$$\phi = RH = \frac{p_\upsilon}{p_{\upsilon s}}$$

$$p_\upsilon = 0.03169x0.7 = 0.02218bar$$

$$H_1 = 1.005t_{db} + \omega_1(2500 + 1.88t_{db})$$
$$= 1.008\text{x}18 + 0.006343(2500 + 1.88\text{x}15)$$
$$= 34.12\text{J/kg of dry air}$$

For air at 25°C and 70% RH, $\text{V} = 35\text{m}^3 / \text{min}$

$$P_{\upsilon s} = 0.03169bar$$

$$\phi = RH = \frac{p_\upsilon}{p_{\upsilon s}}$$

$$p_\upsilon = 0.03169x0.7 = 0.02218bar$$

$$Mass\text{ of air} = \frac{(1.01325\text{ - }0.02218\text{x}10^2\, x35)}{0.287x298}$$
$$\text{m}_2 = 40.55kg.\text{min}$$

$$\omega_2 = \frac{0.622x0.02218}{(1.01325 - 0.02218)} = 0.01392kg / kg\text{ of dry air}$$

$$\text{H}_2 = (1.005x25) + 0.01392(2500 + 1.88x25)$$
$$H_2 = 60.59kJ / kg\text{ of dry air}$$

Mass of dry air/Unit mass of moist air

$$\text{m}_{a1} = \frac{m_1}{1 + \omega_1} = \frac{6.672}{1 + 0.006343} = 6.6299$$

Since $\text{m}_{a2} = \frac{m_2}{1 + \omega_2} = \frac{40.55}{1 + 0.01392} = 39.993$

Then enthalpy of the mixed air,

$$\text{H}_{\text{mix}} = \frac{m_{a1}(H_1) + m_{a2}(H_2)}{m_1 + m_2}$$
$$= \frac{6.6299(34.12) + 39.993(60.56)}{6.672 + 40.55)}$$
$$= 55.96\text{ kJ/kg of dry air}$$

Specific Humidity of the mixed air,

$$\omega_{\text{mix}} = \frac{m_{a1}(\omega_1) + m_{a2}(\omega_2)}{m_1 + m_2}$$

$$= \frac{(6.6299\text{x}0.006343) + (39.993\text{x}0.01932)}{6.672 + 40.55}$$

$$= 0.01268\text{kg/kg of dry air}$$

$$\text{But H}_{\text{mix}} = 1.005t_{db} + \omega_{mix}(2500 + 1.88t_{db})$$

$$55.96 = 1.005xt_{db} + 0.01234(2500 + 1.88t_{db})$$

$$t_{db} = 24.42°C$$

DBT of the mixture = 24.42°C

From chart WBT = 19°C

RH = 67%

Problem 5:

An air conditioning system is designed under the following conditions

Outdoor conditions: 30°CDBT, 75% RH

Required indoor conditions: 22°CDBT,70% RH

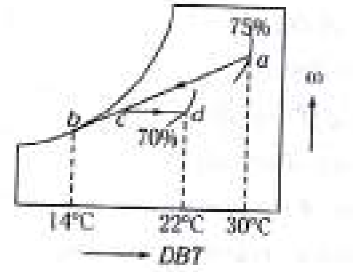
Amount of Free air circulated 3.33 m3/s

Coil dew point temperature DPT=14°

The required condition is achieved first by cooling and dehumidification and then by heating. Estimate

1) The capacity of the cooling coil in tons of refrigeration
2) *Capacity of the heating coil in kW*
3) *The amount of water vapour removed in kg/hr*

Solution:



Locate point 'a' 30°C DBT, 75%RH out door condition

Locate point 'd' 22°C DBT, 70%RH required condition

Locate point 'b' 14°C DPT, coil surface temperature

Join ab

at d, draw a horizontal line to cut the

line ab at point c.

ac → cooling and dehumidification

cd → heating

$$\mathit{From}\ \text{chart}$$

$$\mathrm{H_a} = 83kJ/kg \text{ of air}$$

$$\mathrm{H_b} = 40kJ/kg \text{ of air}$$

$$\mathrm{H_d} = 53kJ/kg \text{ of air}$$

$$\mathrm{H_c} = 48kJ/kg \text{ of air}$$

$$W_a = 0.0202kg/kg \text{ of dry air}$$

$$W_c = W_d = 0.0118kg/kg \text{ of dry air}$$

$$\mathrm{V_{sa}} = 0.88m^3/kg$$

$$\mathit{Mass} \text{ of air} = \frac{V}{V_a} = \frac{3.33}{0.88} = 3.78kg/s$$

$$\text{Capacity of cooling coil} = \frac{\mathrm{m_a}(H_a - H_c)}{3.5}$$

$$= \frac{3.78(83-48)}{3.5} = 37.84\mathit{tons} \text{ of refrigeration}$$

$$\text{Capacity of heating coil} = \mathrm{m_a}(H_d - H_c)$$

$$= 3.78(53\text{-}48) = 18.92\mathrm{kW}$$

$$\mathit{Amount} \text{ of water vapour removed} = \mathrm{m_a}(\omega_a - \omega_d)3600$$

$$= 3.78(0.0202\text{-}0.0118)3600$$

$$= 114.3\text{kg/hr}$$

Problem 6:

A summer air conditioning system for hot and humid weather (DBT=32°Cand 70% RH)

Consists in passing the atmosphere air over a cooling coil where the air is cooled and dehumidified. The air leaving the cooling coil is saturated at the coil temperature. It is then sensibly heated to the required comfort condition of 24°C and 50%RH by passing it over an electric heater then delivered to the room.

Sketch the flow diagram of the arrangement and represent the process undergone by the air on a skeleton psychometric chart and determine

1) *The temperature of the cooling coil*
2) *The amount of moisture removed per kg of dry air in the cooling coil.*
3) *The heat removed per kg of dry air in the cooling coil and*
4) *The heat added per kg of dry air in the heating coil*

$$\mathit{From}\ \text{chart}$$

$$\mathrm{H_a} = 86kJ/kg \text{ of air}$$

$$\mathrm{H_b} = 38kJ/kg \text{ of air}$$

$$\mathrm{H_c} = 48.5kJ/kg \text{ of air}$$

$$\omega_a = 0.021kg/kg \text{ of dry air}$$

$$\omega_b = 0.0092kg/kg \text{ of dry air}$$

$$\text{The temperature of the cooling coil} = T_b = 13°C$$

$$\text{Amount of moisture removed} = \omega_a - \omega_b$$

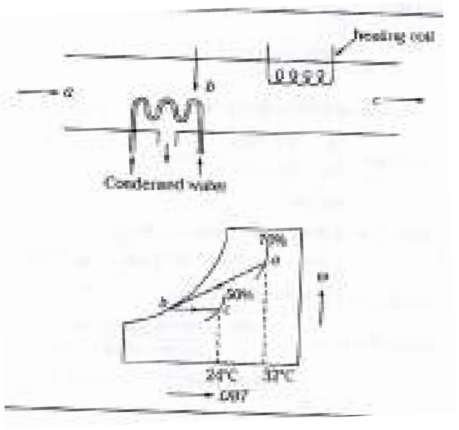
$$= 0.021 - 0.0092 = 0.0108 \text{kg/kg of dry air}$$

$$\text{Heat removed} = H_a - H_b = 86 - 38$$

$$= 48 \text{ kJ/kg of dry air}$$

$$\text{Heat added} = H_c - H_b = 48.5 - 38$$

$$= 10.5 \text{ kJ/kg of dry air}$$



Locate point 'a' 32°C , 70%RH out door condition

Locate point 'c' 24°C DBT, 50%RH required condition

At c draw a horizontal line to cut the saturation line at point 'b'

Join ab

ab → cooling and dehumidification

bc → heating

Problem 7

It is required to design an air conditioning plant for an office room with the following conditions.

Outdoor conditions: 14°CDBT, 10°CWBT

Required conditions: 20°CDBT,60% RH

Amount of air circulated 0.3m3/min/person

Starting capacity of the office= 60

The required condition is achieved first by heating and then by adiabatic humidifying. Determine the following.

Heating capacity of the coil in kW and the surface temperature required, if the by pass factor of the coil is 0.4

Capacity of the humidifier.

Locate point 'a' 14°C , and 10°CWBT (out door condition)

Locate point 'c' 20°C DBT, 60%RH required condition

At a draw a horizontal line

At 'c' draw a constant enthalpy line to cut the horozontal line at point 'b'

Join ab

ab → heating

bc → adiabatic humidification

From chart

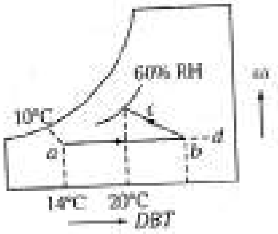
$\mathrm{H_a} = 30kJ/kg$ of air

$\mathrm{H_b} = \mathrm{H_c} = 43kJ/kg$ of air

$\omega_a = \omega_b = 0.006kg/kg$ of dry air

$\omega_c = 0.00875$ kg/kg of dry air

Specific volome $\mathrm{V_{sa}} = 0.8175m^3/kg$



Volume of air supplied $= \mathrm{V} = \dfrac{0.3x60}{60} = 0.3m^3/\sec$

Weight of air supplied $\mathrm{m_a} = \dfrac{V}{V_a} = \dfrac{0.3}{0.8175}$

$= 0.3669\mathrm{kg/sec}$

Capacity of the heating coil $= \mathrm{m_a}(H_b - H_a)$

$= 0.3669(43 - 30) = 4.77\mathrm{kW}$

From chart $\mathrm{T_b} = 26.5°C$

Let coil surface temperature be $\mathrm{T_d}$

By passing factor $= \dfrac{\mathrm{T_d} - T_b}{\mathrm{T_d} - T_a} = 0.4T_d - 5.6 = T_d - 26kJ$

$$0.4 = \frac{\mathrm{T_d} - 26.5}{\mathrm{T_d} - 1.4}$$

$$T_d = 34.8°C$$

Capacity of the humidifier $= \mathrm{m_a}(\omega_c - \omega_b)x3600$

$= 0.3669(0.00875 - 0.006)3600$

$= 3.63\mathrm{kg/hour}$

Problem 8

An air conditioned system is to be designed for a hall of 200 seating capacity when the following conditions are given:

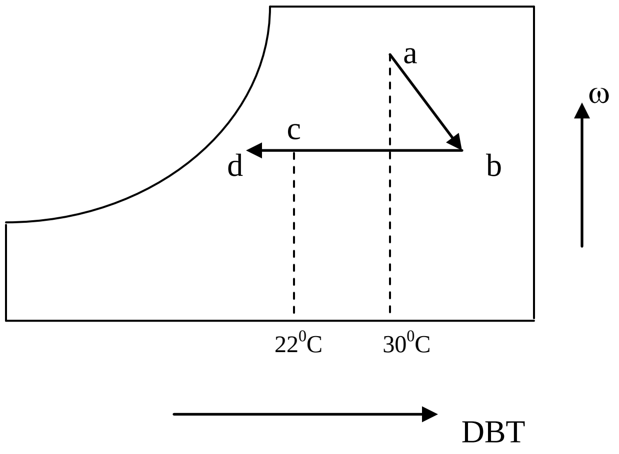
Atmospheric condition = 30^0C DBT and 50% RH
Indoor condition = 22^0C DBT and 60% RH
Volume of air required = 0.4m^3/min/person

The required condition is achieved first by chemical dehumidification and after that by sensible cooling.

Find the following .

a) DBT of the air leaving the dehumidifier.
b) The quantity of water vapour removed in the duhumidifier per hour.
c) The capacity of cooling coil in tons of refrigeration.
d) Surface temperature of the coil if the by pass factor of the coil is 0.25.

Solution:



Locate point 'a', 30^0C DBT, 50% RH, the atmospheric condition.
Locate point 'c', 22^0C DBT, 60% RH, the required indoor condition.
"Since chemical dehumidification process follows constant enthalpy line"
at a draw a line parallel to constant enthalpy line.
At 'c' draw a constant ω line to cut the previous line at point b.

a) DBT of air leaving the dehumidifier $T_b = 40.5^0C$

From chart

Hb = Ha = 65kJ/kg, $\omega_a = 0.013$ kg/kg of dry air

Hc = 45 kJ/kg, $\omega_b = 0.009$ kg/kg of dry air

$V_{sa} = 0.875\ m^3/min$

Volume of air = 200 X 0.4 = 80 m^3/min

W_a = Weight of air = V/Vsa = 80/0.875 = 91.42 kg/min

b) Quantity of water vapour removed/hour = $W_a(\omega_a-\omega_b)60$
= 91.42(0.13-0.009)60 = 21.94 kg/hr

c) Capacity of cooling coil = $W_a(H_a-H_b)$/ (60 X 3.5) = 91.42(65-45)/(60 X 3.5)
= 8.7 tons

d) By pass factor = $(T_c-T_d)/(T_b-T_d) = 0.25$

T_d = Temperature of cooling coil = 15.83^0C

Problem 9

An air conditioned system is to be designed for a cinema hall of 1000 seating capacity when the following conditions are given:
Outdoor condition = 11^0C DBT and 70% RH
Required indoor condition = 20^0C DBT and 60% RH

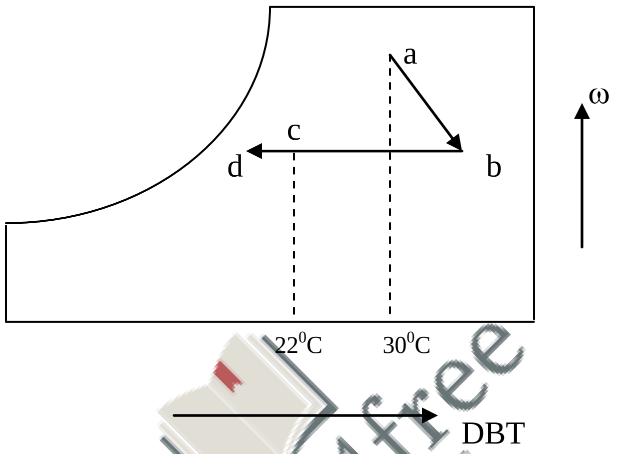
Amount of air required = 0.3m^3/min/person

The required condition is achieved first by heating, then by humidifuing and finaly by heating. The condition of air coming out of the humidifier is 75% RH.

Find the following .

a) Heating capacity of the first heater in kW and condition of the air coming out of the first heater in kW and condition of the air

Solution:



Locate point 'a', 30^0C DBT, 50% RH, the atmospheric condition.

Locate point 'c', 22^0C DBT, 60% RH, the required indoor condition.

"Since chemical dehumidification process follows constant enthalpy line"

at a draw a line parallel to constant enthalpy line.

At 'c' draw a constant ω line to cut the previous line at point b.

e) DBT of air leaving the dehumidifier $T_b = 40.5^0C$

From chart

Hb = Ha = 65kJ/kg, $\omega_a = 0.013$ kg/kg of dry air

Hc = 45 kJ/kg, $\omega_b = 0.009$ kg/kg of dry air

$V_{sa} = 0.875\ m^3/min$

Volume of air = 200 X 0.4 = 80 m^3/min

W_a = Weight of air = V/Vsa = 80/0.875 = 91.42 kg/min

f) Quantity of water vapour removed/hour = $W_a(\omega_a-\omega_b)60$

= 91.42(0.13-0.009)60 = 21.94 kg/hr

g) Capacity of cooling coil = $W_a(H_a-H_b)$/ (60 X 3.5) = 91.42(65-45)/(60 X 3.5)

= 8.7 tons

h) By pass factor = $(T_c-T_d)/(T_b-T_d) = 0.25$

T_d = Temperature of cooling coil = 15.83^0C

Reciprocating Compressor

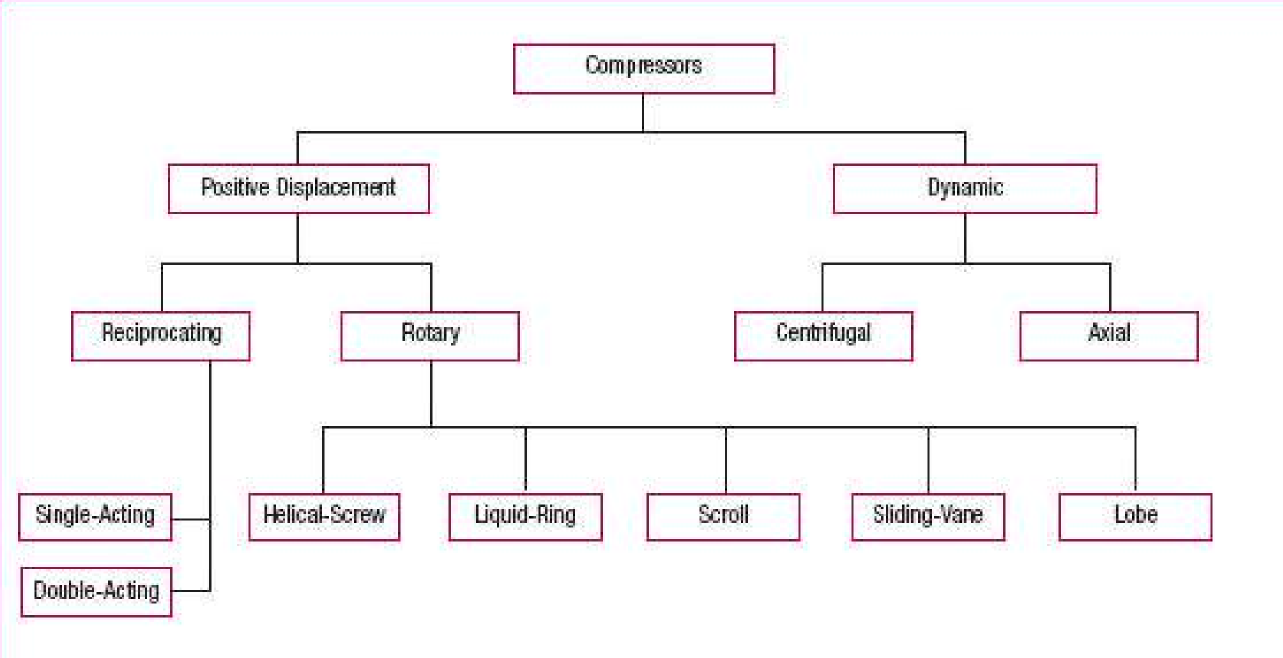
5.1 INTRODUCTION

Compressors are work absorbing devices which are used for increasing pressure of fluid at the expense or work done on fluid.

The compressors used for compressing air are called air compressors. Compressors are invariably used for all applications requiring high pressure air. Some of popular applications of compressor are, for driving pneumatic tools and air operated equipments, spray painting, compressed air engine, supercharging surface cleaning, refrigeration and air conditioning, chemical industry etc. compressors are supplied with low pressure air (or any fluid) at inlet which comes out as high pressure air (or any fluid) at outlet. Work required for increasing pressure of air is available from the prime mover driving the compressor. Generally, electric motor, internal combustion engine or steam engine, turbine etc. are used as prime movers. Compressors are similar to fans and blowers but differ in terms of pressure ratios. Fan is said to have pressure ratio up to 1.1 and blowers have pressure ratio between 1.1 to 4 while compressors have pressure ratios more than 4.

5.2 CLASSIFICATION OF COMPRESSORS

Table-5.1 Types of Compressors



Compressors can be classified in the following different ways.

(a) **Based on principle of operation:** Based on the principle of operation compressors can be classified as.

  - (i) Positive displacement compressor.
  - (ii) Non-positive displacement compressors.

In positive displacement compressors the compression is realized by displacement of solid boundary and preventing fluid by solid boundary from flowing back in the direction of pressure gradient. Due to solid wall displacement these are capable of providing quite large pressure ratios. Positive displacement compressors can be further classified based on the type of mechanism used for compression. These can be

(i) Reciprocating type positive displacement compressors
(ii) Rotary type positive displacement compressors.

Reciprocating compressors generally, employ piston-cylinder arrangement where displacement of piston in cylinder causes rise in pressure. Reciprocating compressors are capable of giving large pressure ratios but the mass handling capacity is limited or small. Reciprocating compressors may also be single acting compressor or double acting compressor. Single acting compressor has one delivery stroke per revolution while in double acting there are two delivery strokes per revolution of crank shaft. Rotary compressors employing positive displacement have a rotary part whose boundary causes positive displacement of fluid and thereby compression. Rotary compressors of this type are available in the names as given below;

(i) Roots blower
(ii) Vane type compressors

Rotary compressors of above type are capable of running at higher speed and can handle large mass flow rate than reciprocating compressors of positive displacement type.

Non-positive displacement compressors, also called as steady flow compressors use dynamic action of solid boundary for realizing pressure rise. Here fluid is not contained in definite volume and subsequent volume reduction does not occur as in case of positive displacement compressors. Non-positive displacement compressor may be of ‘axial flow type’ or ‘centrifugal type’ depending upon type of flow in compressor.

(b) **Based on number of stages:** Compressors may also be classfied on the basis of number of stages. Generally, the number of stages depend upon the maximum delivery pressure. Compressors can be single stage or multistage. Normally maximum compression ratio of 5 is realized in single stage compressors. For compression ratio more than 5 the multistage compressors are used.

Type values of maximum delivery pressures generally available from different type of compressor are,

(i) Single stage Compressor, for delivery pressure upto 5 bar.
(ii) Two stage Compressor, for delivery pressure between 5 to 35 bar
(iii) Three stage Compressor, for delivery pressure between 35 to 85 bar.
(iv) Four stage compressor, for delivery pressure more than 85 bar

(c) **Based on Capacity of compressors** : Compressors can also be classified depending upon the capacity of Compressor or air delivered per unit time. Typical values of capacity for different compressors are given as;

(i) Low capacity compressors, having air delivery capacity of 0.15 m^3/s or less
(ii) Medium capacity compressors, having air delivery capacity between 0.15 to 5 m^3/s.
(iii) High capacity compressors, having air delivery capacity more than 5 m^3/s

(d) Based on highest pressure developed: Depending upon the maximum pressure available from compressor they can be classified as low pressure, medium pressure, high pressure and super high pressure compressors. Typical values of maximum pressure developed for different compressors are as under:

(i) Low pressure compressor, having maximum pressure upto 1 bar
(ii) Medium pressure compressor, having maximum pressure from 1 bar to 8 bar
(iii) High pressure compressor, having maximum pressure from 8 to 10 bar
(iv) Super high pressure compressor, having maximum pressure more than 10 bar.

5.3 Reciprocating Compressors

Reciprocating Compressor has piston cylinder arrangement as shown Fig.5.1

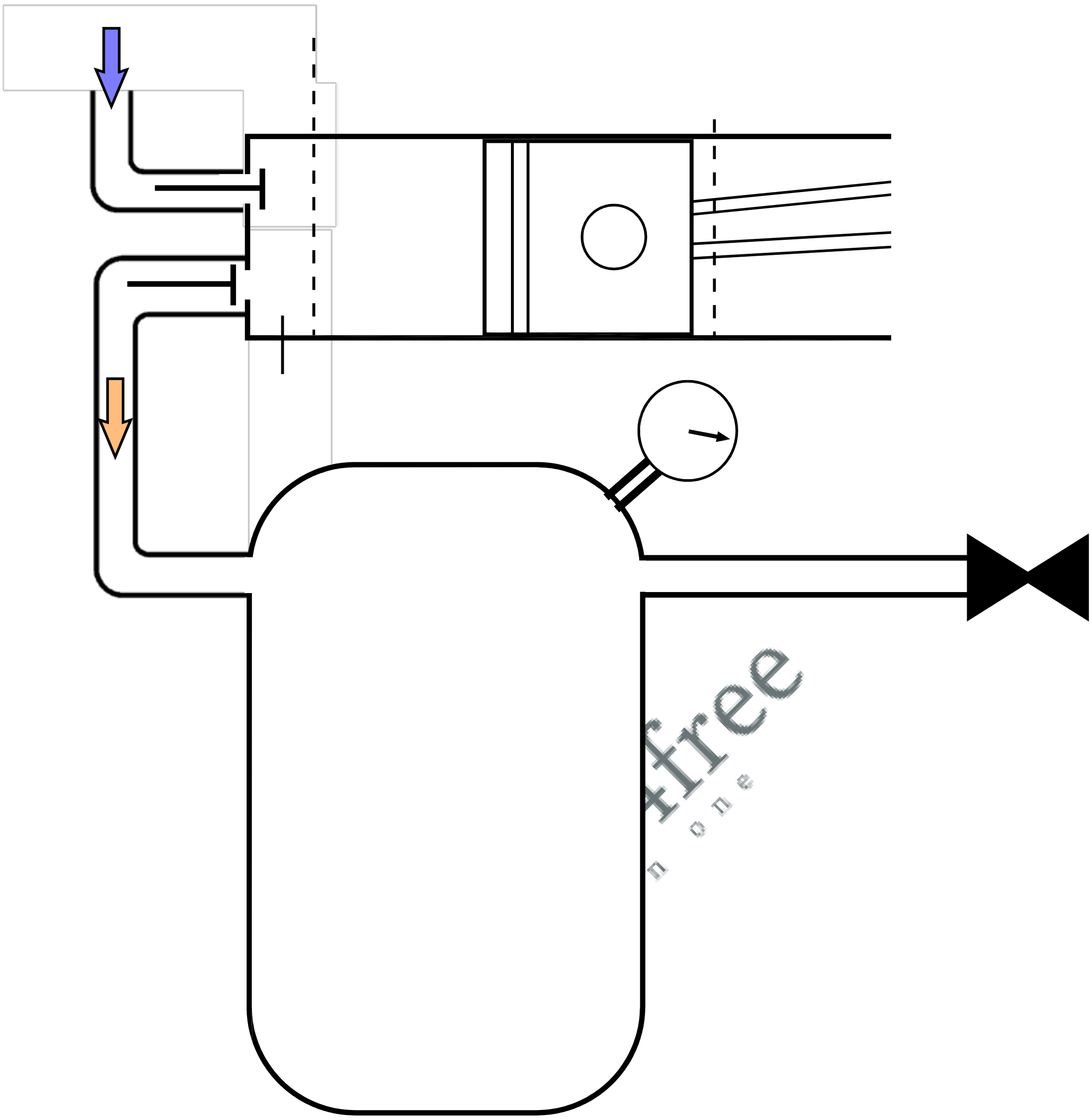


Fig.5.1 Reciprocating Compressor

Reciprocating Compressor has piston, cylinder, inlet valve, exit valve, connecting rod, crank, piston pin, crank pin and crank shaft. Inlet valve and exit valves may be of spring loaded type which get opened and closed due to pressure differential across them. Let us consider piston to be at top dead centre (TDC) and move towards bottom dead centre (BDC). Due to this piston movement from TDC to BDC suction pressure is created causing opening of inlet valve. With this opening of inlet valve and suction pressure the atmospheric air enters the cylinder.

Air gets into cylinder during this stroke and is subsequently compressed in next stroke with both inlet valve and exit valve closed. Both inlet valve and exit valves are of plate type and spring loaded so as to operate automatically as and when sufficient pressure difference is available to cause deflection in spring of valve plates to open them. After piston reaching BDC it reverses its motion and compresses the air inducted in previous stroke. Compression is continued till the pressure of air inside becomes sufficient to cause deflection in exit valve. At the moment when exit valve plate gets lifted the exhaust of compressed air takes place. This piston again reaches TDC from where downward piston movement is again accompanied by suction. This is how reciprocating compressor. Keeps on working as flow device. In order to counter for the heating of piston-cylinder arrangement during compression the provision of cooling the cylinder is there in the form of cooling jackets in the body. Reciproting compressor described above has suction, compression and discharge as three prominent processes getting compelted in two strokes of piston or one revolution of crank shaft.

5.4 Thermodynamic Analysis

Compression of air in compressor may be carried out following number of thermodynamic processes such as isothermal compression, polytropic compressor or adiabatic compressor. Fig.16.3 shows the thermodynamic cycle involved in compressor. Theoretical cycle is shown neglecting clearance volume but in actual cycle clearance volume can not be negligible. Clearance volume is necessary in order to prevent collision of piston with cylinder head, accommodating valve mechanism etc., Compression process is shown by process1-2, 1-2^{1}, 1-2^{11} and 1-2^{111} following isothermal, polytropic and adiabatic processes.

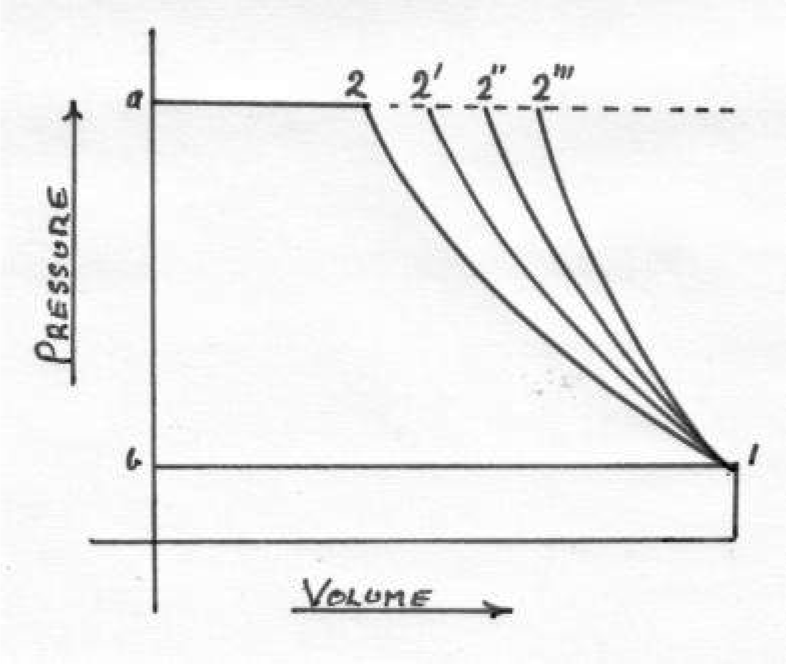


Fig.5.2 P-V diagram for Reciprocating Compressor without Clearance

On P-V diagram process 4-1 shows the suction process followed by compression during 1-2 and discharge through compressor is shown by process 2-3.

Air enters compressor at pressure p_1 and is compressed upto p_2. Compression work requirement can be estimated from the area below the each compression process. Area on p-V diagram shows that work requirement shall be minimum with isothermal process 1-2”. Work requirement is maximum with process 1-2 ie., adiabatic process. As a designer one shall be interested in a compressor having minimum compression work requirement. Therefore, ideally compression should occur isothermally for minimum work input. In practice it is not possible to have isothermal compression because constancy of temperature during compression can not be realized. Generally, compressors run at substantially high speed while isothermal compression requires compressor to run at very slow speed so that heat evolved during compression is dissipated out and temperature remains constant. Actually due to high speed running of compressor the compression process may be assumed to be near adiabatic or polytropic process following law of compression as $pV^n=C$ with of ‘n’ varying between 1.25 to 1.35 for air. Compression process following three processes is also shown on T-s diagram in Fig.16.4. it is thus obvious that actual compression process should be compared with isothermal compression process. A mathematical parameter called isothermal efficiency is defined for quantifying the degree of deviation of actual compression process from ideal compression process. Isothermal efficiency is defined by the ratio is isothermal work and actual indicated work in reciprocating compressor.

$$\text{Isothermal efficiency} \;=\; \frac{\text{Isothermal work}}{\text{Actual indicated Work}}$$

Practically, compression process is attempted to be closed to isothermal process by air/water cooling, spraying cold water during compression process. In case of multistage compression process the compression in different stages is accompanied by intercooling in between the stages. P_2 V_2

Mathematically, for the compression work following polytropic process, $PV^n=C$. Assuming negligible clearance volume the cycle work done.

Wc = Area on p-V diagram

$$\text{Wc} = \left[p_2V_2 + \left(\frac{p_2V_2 - p_1V_1}{n-1} \right) \right] - p_1V_1 \qquad 1$$

$$= \left[\left(\frac{n}{n-1} \right) [p_2V_2 - p_1V_1] \right]$$

$$= \left(\frac{n}{n-1} \right) (p_1V_1) \left[\frac{p_2V_2}{p_1V_1} - 1 \right]$$

$$= \left(\frac{n}{n-1} \right) (p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\left(\frac{n-1}{n} \right)} - 1 \right]$$

$$= \left(\frac{n}{n-1} \right) (mRT_1) \left[\left(\frac{p_2}{p_1} \right)^{\left(\frac{n-1}{n} \right)} - 1 \right]$$

$$= \left(\frac{n}{n-1} \right) (mR)(T_2 - T_1)$$

In case of compressor having isothermal compression process, n = 1, ie., $p_1V_1 = p_2V_2$

$$W_{iso} = p_2V_2 + p_1V_1 \ln r - p_1V_1$$

$$W_{iso} = p_1V_1 \ln r, \qquad where, r = \frac{V_1}{V_2}$$

In case of compressor having adiabatic compression process,

$$W_{adiabatic} = \left(\frac{\gamma}{\gamma - 1} \right) (mR)(T_2 - T_1)$$

Or

$$W_{adiabatic} = (mC_p)(T_2 - T_1)$$

$$W_{adiabatic} = (m)(h_2 - h_1)$$

$$\eta_{iso} = \frac{p_1 V_1 \ln r}{\left(\frac{n}{n-1}\right)(p_1 V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]}$$

The isothermal efficiency of a compressor should be close to 100% which means that actual compression should occur following a process close to isothermal process. For this the mechanism be derived to maintain constant temperature during compression process. Different arrangements which can be used are:

(i) Faster heat dissipation from inside of compressor to outside by use of fins over cylinder. Fins facilitate quick heat transfer from air being compressed to atmosphere so that temperature rise during compression can be minimized.

(ii) Water jacket may be provided around compressor cylinder so that heat can be picked by cooling water circulating through water jacket. Cooling water circulation around compressor regulates rise in temperature to great extent.

(iii) The water may also be injected at the end of compression process in order to cool the air being compressed. This water injection near the end of compression process requires special arrangement in compressor and also the air gets mixed with water and needs to be separated out before being used. Water injection also contaminates the lubricant film inner surface of cylinder and may initiate corrosion etc, The water injection is not popularly used.

(iv) In case of multistage compression in different compressors operating serially, the air leaving one compressor may be cooled upto ambient state or somewhat high temperature before being injected into subsequent compressor. This cooling of fluid being compressed between two consecutive compressors is called intercooling and is frequently used in case of multistage compressors.

Considering clearance volume: With clearance volume the cycle is represented on Fig.5.3 The work done for compression of air polytropically can be given by the are a enclosed in cycle 1-2-3-4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.

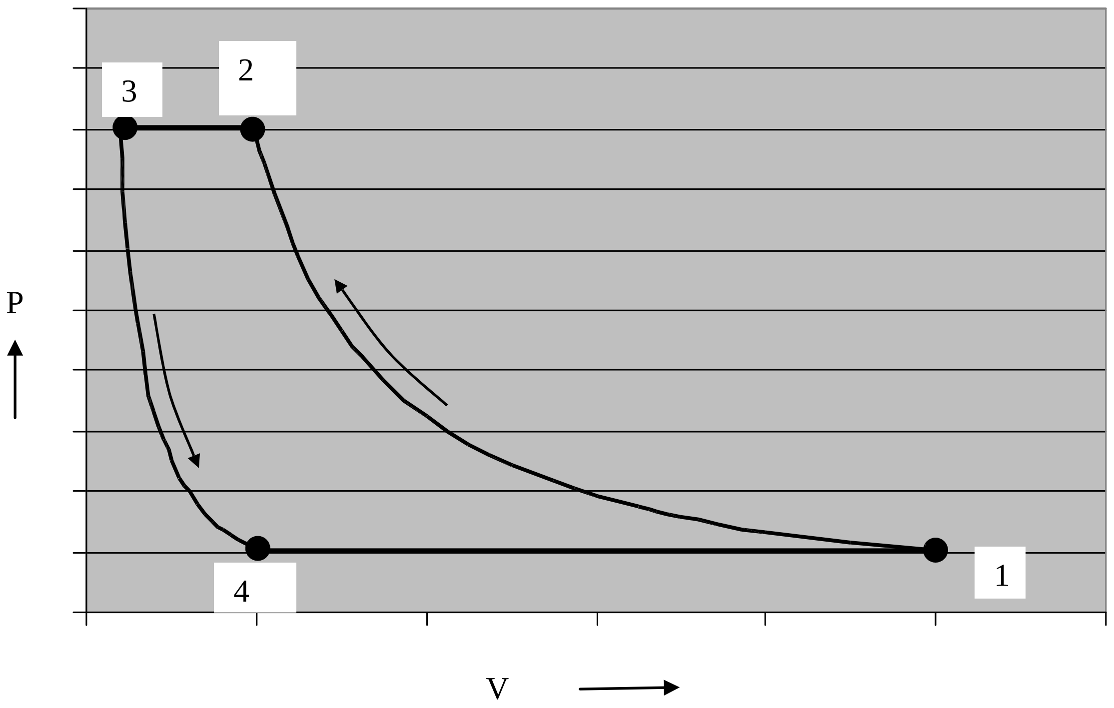


Fig.5.3 P-V diagram for Reciprocating Compressor with Clearance

$W_{c,\text{with CV}}$ = Area 1234

$$=\left(\frac{n}{n-1}\right)\left(p_1V_1\right)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}-1\right]-\left(\frac{n}{n-1}\right)\left(p_4V_4\right)\left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}}-1\right]$$

Here $P_1 = P_4$, $P_2 = P_3$

$$=\left(\frac{n}{n-1}\right)\left(p_1V_1\right)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}-1\right]-\left(\frac{n}{n-1}\right)\left(p_1V_4\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$=\left(\frac{n}{n-1}\right)\left(p_1\right)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}-1\right]\left(V_1-V_4\right)$$

In the cylinder of reciprocating compressor (V_1-V_4) shall be the actual volume of air delivered per cycle. $V_d = V_1 – V_4$. This ($V_1 – V_4$) is actually the volume of air in hated in the cycle and delivered subsequently.

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(p_1 V_d)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right]$$

If air is considered to behave as perfect gas then pressure, temperature, volume and mass can be inter related using perfect gas equation. The mass at state 1 may be given as m_1 mass at state 2 shall be m1, but at state 3 after delivery mass reduces to m_2 and at state 4 it shall be m_2.

So, at state 1, $p_1 V_1 = m_1 R T_1$

at state 2, $p_2 V_2 = m_1 R T_2$

at state 3, $p_3 V_3 = m_2 R T_3$ or $p_2 V_3 = m_2 R T_3$

at state 4, $p_4 V_4 = m_2 R T_4$ or $p_1 V_4 = m_2 R T_4$

Ideally there shall be no change in temperature during suction and delivery
i.e., $T_4 = T_1$ and $T_2 = T_3$ from earlier equation

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(p_1)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right](V_1 - V_4)$$

Temperature and pressure can be related as,

$$\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = \frac{T_2}{T_1} \quad \text{and} \quad \left(\frac{p_4}{p_3}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3} \Rightarrow \left(\frac{p_1}{p_2}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3}$$

Substiuting

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 R T_1 - m_2 R T_4)\left[\frac{T_2}{T_1} - 1\right]$$

Substituting for constancy of temperature during suction and delivery.

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 R T_1 - m_2 R T_1)\left[\frac{T_2 - T_1}{T_1}\right]$$

Or

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 - m_2) R (T_2 - T_1)$$

Thus (m_1-m_2) denotes the mass of air sucked or delivered. For unit mass of air delivered the work done per kg of air can be given as,

$$W_{c,withCV} = \left(\frac{n}{n-1}\right) R(T_2 - T_1) \quad \text{per kg of air}$$

Thus from above expressions it is obvius that the clearance volume reduces the effective swept volume i.e., the masso f air handled but the work done per kg of air delivered remains unaffected.

From the cycle work estimated as above the theoretical power required for running compressor shall be,

For single acting compressor running with N rpm, power input required, assuming clearance volume.

$$Powerrequired = \left[\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right] p_1(V_1 - V_4)\right](N)$$

For double acting compressor, Power

$$Powerrequired = \left[\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} - 1\right] p_1(V_1 - V_4)\right](2N)$$

Volumetric efficiency: Volumetric efficiency of compressor is the measure of the deviation from volume handling capacity of compressor. Mathematically, the volumetric efficiency is given by the ratio of actual volume of air sucked and swept volume of cylinder. Ideally the volume of air sucked should be equal to the swept volume of cylinder, but it is not so in actual case. Practically the volumetric efficiency lies between 60 to 90%.

Volumetric efficiency can be overall volumetric efficiency and absolute volumetric efficiency as given below.

$$\text{Overall volumetric efficiency} = \frac{\text{Volume of free air sucked in cylinder}}{\text{Swept volume of LP cylinder}}$$

$$(\text{Volumetric efficiency})_{\text{freeaircondition}} = \frac{\text{Volume of free air sucked in cylinder}}{(\text{Swept volume of LP cylinder})_{freeaircondition}}$$

Here free air condition refers to the standard conditions. Free air condition may be taken as 1 atm or 1.01325 bar and 15°C or 288K. consideration for free air is necessary as otherwise the different compressors can not be compared using volumetric efficiency because specific volume or density of air varies with altitude. It may be seen that a compressor at datum level (sea level) shall deliver large mass than the same compressor at high altitude.

This concept is used for giving the capacity of compressor in terms of 'free air delivery' (FAD). "Free air delivery is the volume of air delivered being reduced to free air conditions". In case of air the free air delivery can be obtained using perfect gas equation as,

$$\frac{p_aV_a}{T_a} = \frac{p_1(V_1 - V_4)}{T_1} = \frac{p_2(V_2 - V_3)}{T_2}$$

Where subscript a or p_a, V_a, T_a denote properties at free air conditions

$$V_a = \frac{p_1T_a}{p_a}\frac{p_1(V_1 - V_4)}{T_1} = \text{ FAD per cycle}$$

This volume V_a gives 'free air delivered' per cycle by the compressor.

Absolute volumetric efficiency can be defined, using NTP conditions in place of free air conditions.

$$\eta_{vol} = \frac{FAD}{Sweptvolume} = \frac{V_a}{(V_1 - V_2)} = \frac{p_1T_a(V_1 - V_4)}{p_aT_1(V_1 - V_3)}$$

$$\eta_{vol} = \left(\frac{p_1T_a}{p_aT_1}\right)\left\{\frac{(V_s + V_c) - V_4}{V_s}\right\}$$

Here V_s is the swept volume = $V_1 - V_3$

V_c is the clearance volume = V_3

$$\eta_{vol} = \left(\frac{p_1T_a}{p_aT_1}\right)\left\{1 + \left(\frac{V_c}{V_s}\right) - \left(\frac{V_4}{V_s}\right)\right\}$$

Here $\frac{V_4}{V_s} = \frac{V_4}{V_c}\cdot\frac{V_c}{V_s} = \left(\frac{V_4}{V_3}\cdot\frac{V_c}{V_s}\right)$

Let the ratio of clearance volume to swept volume be given by C. = $\frac{V_c}{V_s}$

$$\eta_{vol} = \left(\frac{p_1T_a}{p_aT_1}\right)\left\{1 + C - C\left(\frac{V_4}{V_3}\right)\right\}$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1}\right)\left\{1 + C - C\left(\frac{p_2}{p_1}\right)^{1/n}\right\}$$

Volumetric efficiency depends on ambient pressure and temperature, suction pressure and temperature, ratio of clearance to swept volume, and pressure limits. Volumetric efficiency increases with decrease in pressure ratio in compressor.

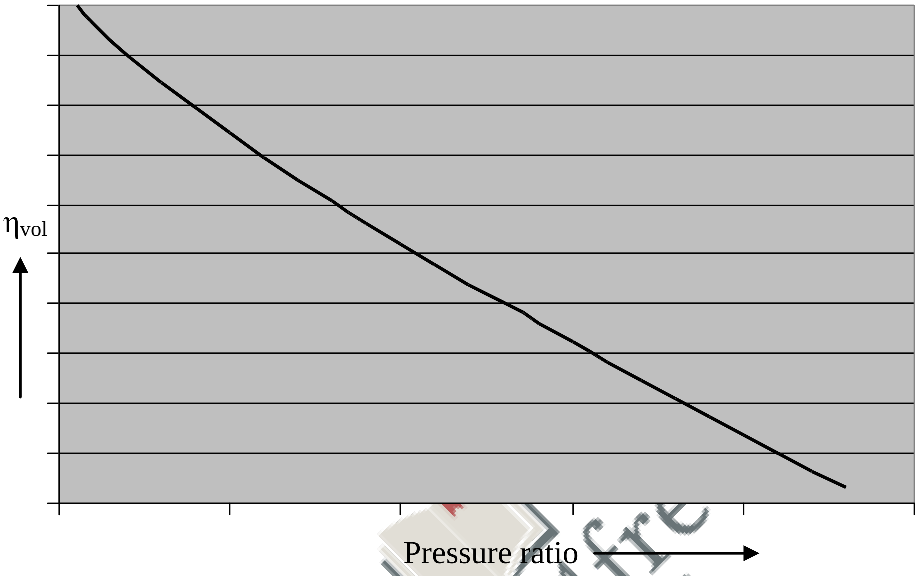


Fig.5.4 Volumetric efficiency v/s Pressure ratio

Multistage Compression

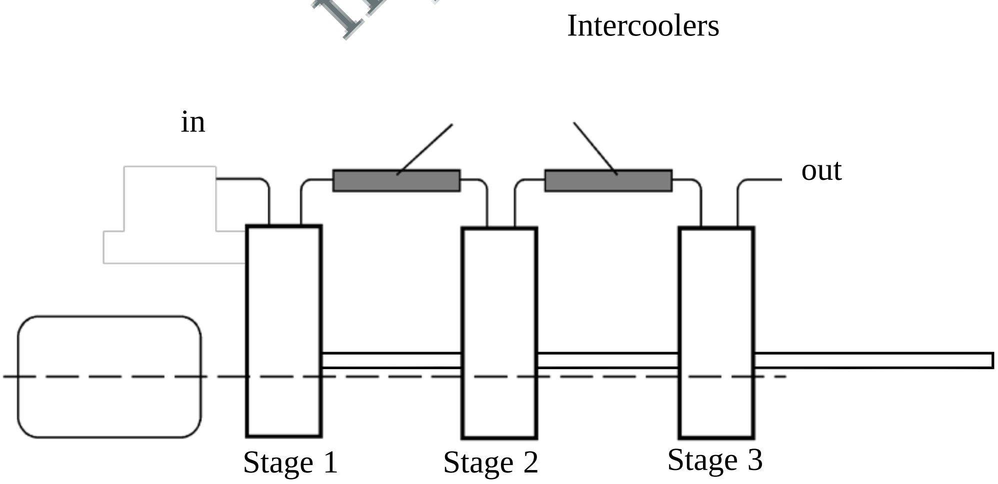


Fig.5.5 Multistage Compressor with inter coolers

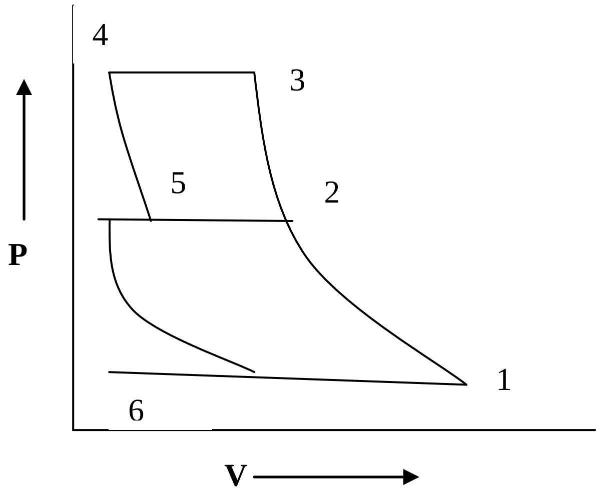


Fig.5.6 P-V diagram for Multistage Compressor

Multistage compression refers to the compression process completed in more than one stage i.e., a part of compression occurs in one cylinder and subsequently compressed air is sent to subsequent cylinders for further compression. In case it is desired to increase the compression ratio of compressor then multi-stage compression becomes inevitable. If we look at the expression for volumetric efficiency then it shows that the volumetric efficiency decreases with increase in pressure ratio. This aspect can also be explained using p-V representation shown in Fig.5.6.

Therefore, the volumetric efficiency reduces with increasing pressure ratio in compressor with single stage compression. Also for getting the same amount of free air delivery the size of cylinder is to be increased with increasing pressure ratio. The increase in pressure ratio also requires sturdy structure from mechanical strength point of view fro withstanding large pressure difference.

The solution to number of difficulties discussed above lies in using the multistage compression where compression occurs in parts in different cylinders one after the other. Fig.16.6b, shows the multistage compression occurring in two stages. Here first stage of compression occurs in cycle 12671 and after first stage compression partly compressed enters second stage of compression and occurs in cycle 2345. In case of multistage compression the compression in first stage occurs at low temperature and subsequent compression in following stages occurs at higher temperature. The compression work requirement depends largely upon the average temperature during compression. Higher average temperature during compression has larger work requirement compared to low temperature so it is always desired to keep the low average temperature during compression.

Apart from the cooling during compression the temperature of air at inlet to compressor can be reduced so as to reduce compression work. In multistage compression the partly compressed air leaving first stage is cooled upto ambient air temperature in intercooler and then sent to subsequent cylinder (stage) for compression. Thus, intercoolers when put between the stages reduce the compression work and compression is called intercooled compression. Intercooling is called perfect when temperature at inlet to subsequent stages of compression is reduced to ambient temperature. Fig.16.6c, shows multi-stage (two stage) intercooled compression. Intercooling between two stages causes temperature drop from 2 to 2’ i.e discharge from first stage (at 2) is cooled upto the ambient temperature stage (at2’) which lies on isothermal compression process 1-2’-3”. In the absence of intercooling the discharge from first stage shall enter at 2. Final discharge from second stage occurs at 3’ in case of intercooled compression compared to discharge at 3 in case of non-intercooled compression. Thus, intercooling offers reduced work requirement by the amount shown by area 22’3’3 on p-V diagram. If the intercooling is not perfect then the inlet state to second/subsequent stage shall not lie on the isothermal compression process line and this stage shall lie between actual discharge state from first stage and isothermal compression line.

Fig.16.7 shows the schematic of multi stage compressor (double stage) with inter cooler between stage T-s representation is shown in Fig.16.8. The total work requirement for running this shall be algebraic summation of work required for low pressure (LP) and high pressure (HP) stages. The size of HP cylinder is smaller than LP cylinder as HP cylinder handles high pressure air having smaller specific volume.

Mathematical analysis of multistage compressor is done with following assumptions:

(i) Compression in all the stages is done following same index of compression and there is no pressure drop in suction and delivery pressures in each stage. Suction and delivery pressure remains constant in the stages.

(ii) There is perfect intercooling between compression stages.

(iii) Mass handled in different stages is same i.e, mass of air in LP and HP stages are same.

(iv) Air behaves as perfect gas during compression.

From combined p-V diagram the compressor work requirement can be given as,

Work requirement in LP cylinder, $W_{LP} = \left(\frac{n}{n-1}\right)P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$

Work requirement in HP cylinder, $W_{HP} = \left(\frac{n}{n-1}\right)P_2V_2\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$

For perfect intercooling, $p_1V_1 = p_2V_2$’ and

$$W_{HP} = \left(\frac{n}{n-1}\right)P_2V_{2'}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$$

Therefore, total work requirement, $W_c = W_{LP} + W_{HP}$, for perfect inter cooling

$$W_C = \left(\frac{n}{n-1}\right)\left[P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right\} + P_2V_{2'}\left\{\left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$= \left(\frac{n}{n-1}\right)\left[P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right\} + P_1V_1\left\{\left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$W_C = \left(\frac{n}{n-1}\right)P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_{2'}}{P_1}\right)^{\frac{n-1}{n}} - 2\right]$$

Power = W_c X N - Watts

If we look at compressor work then it shows that with the initial and final pressures p_1 and $P_{2'}$ remaining same the intermediate pressure p_2 may have value floating between p_1 and $P_{2'}$ and change the work requirement Wc. Thus, the compressor work can be optimized with respect to intermediate pressure P_2. Mathematically, it can be differentiated with respect to P_2.

$$\frac{dW_C}{dP_2} = \frac{d}{dP_2}\left[\left(\frac{n}{n-1}\right)P_1V_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 2\right\}\right]$$

$$\frac{dW_C}{dP_2} = \left[\left(\frac{n}{n-1}\right)P_1V_1\frac{d}{dP_2}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_{2'}}{P_2}\right)^{\frac{n-1}{n}} - 2\right\}\right]$$

$$\frac{dW_C}{dP_2} = \left(\frac{n}{n-1}\right)P_1V\left\{\left(\frac{n-1}{n}\right)P_1^{\frac{1-n}{n}} \cdot P_2^{\frac{-1}{n}} - \left(\frac{n-1}{n}\right) \cdot P_{2'}^{\frac{1-n}{n}} \cdot P_2^{\frac{1-2n}{n}}\right\}$$

Equating to zero yields

$$P_1^{\frac{1-n}{n}} \cdot P_2^{\frac{-1}{n}} = \cdot P_{2'}^{\frac{1-n}{n}} \cdot P_2^{\frac{1-2n}{n}}$$

$$P_2^{\frac{-2+2n}{n}} = \cdot P_{2'}^{\frac{1-n}{n}} \cdot P_1^{\frac{n-1}{n}}$$

$$P_2^{\,2\left(\frac{n-1}{n}\right)} = \left(P_1 \cdot P_{2'}\right)^{\left(\frac{n-1}{n}\right)}$$

$$P_2^{\,2} = \left(P_1 \cdot P_{2'}\right), P_2 = \sqrt{P_1 \cdot P_{2'}}$$

Pressure ratio in Ist stage = Pressure ratio in IInd stage

Thus, it is established that the compressor work requirement shall be minimum when the pressure ratio in each stage is equal.

In case of multiple stages, say *i* number of stages, for the delivery and suction pressures of P_{i+1} and P_1 the optimum stage pressure ratio shall be,

Optimum stage pressure ratio = $\left(\frac{P_{i+1}}{P_1}\right)^{\frac{1}{i}}$ for pressure at stages being P_1, P_2, P_3,……P_{i-1}, P_i, P_{i+1}

Minimum work required in two stage compressor can be given by

$$W_{C,\min} = \left(\frac{n}{n-1}\right)P_1V_1 \cdot 2\left\{\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$$

For *i* number of stages, minimum work,

$$W_{C,\min} = i \cdot \left(\frac{n}{n-1}\right)P_1V_1\left\{\left(\frac{P_{i+1}}{P_i}\right)^{\frac{(n-1)}{n \cdot i}} - 1\right\}$$

It also shows that for optimum pressure ratio the work required in different stages remains same for the assumptions made for present analysis. Due to pressure ration being equal in all stages the temperature ratios and maximum temperature in each stage remains same for perfect intercooling.

Cylinder dimensions: In case of multistage compressor the dimension of cylinders can be estimated basing upon the fact that the mass flow rate of air across the stages remains same. For perfect intercooling the temperature of air at suction of each stage shall be same.

If the actual volume sucked during suction stroke is V_1, V_2, V_3. . . . for different stages they by perfect gas law, $P_1 V_1 = RT_1$, $P_2 V_2 = RT_2$, Pc, $V_3 = RT_3$
For perfect intercooling

$P_1 V_1 = RT_1$, $p_2 V_2 = RT_1$, p_3, $V_3 = RT_1$

$P_1 V_1 = P_2 V_2 = RT_2$, P_3, $V_3 =$ . . . . .

If the volumetric efficiency of respective stages in $\eta_{V_1}, \eta_{V_2}, \eta_{V_3}, \ldots.$

Then theoretical volume of cylinder1, $V_{1,th} = \dfrac{V_1}{\eta_{V_1}}; V_1 = \eta_{V_1} \cdot V_{1,th}$

Cylinder 2, $V_{2,th} = \dfrac{V_1}{\eta_{V_1}}; V_2 = \eta_{V_2} \cdot V_{2,th}$

Cylinder 3, $V_{3,th} = \dfrac{V_3}{\eta_{V_3}}; V_3 = \eta_{V3} \cdot V_{3,th}$

Substituting,
$P_1 \cdot \eta_{V_1} \cdot V_{1,th} = P_2 \cdot \eta_{V_2} \cdot V_{2,th} = P_3 \cdot \eta_{V_3} \cdot V_{3,th} = \ldots$
Theoretical volumes of cylinder can be given using geometrical dimensions of cylinder as diameters D_1, D_2, D_3 . . . and stroke lengths L_1, L_2, L_3 . . . .

Or $\quad V_{1,th} = \dfrac{\pi}{4} \cdot D_1^{\,2} \cdot L_1$

$$V_{2,th} = \frac{\pi}{4} \cdot D_2^{\,2} \cdot L_2$$

$$V_{3,th} = \frac{\pi}{4} \cdot D_3^{\,2} \cdot L_3$$

Or $\quad P_1 \cdot \eta_{Vi} \cdot \dfrac{\pi}{4} \cdot D_1^{\,2} \cdot L_1 = P_2 \cdot \eta_{V2} \cdot \dfrac{\pi}{4} \cdot D_2^{\,2} \cdot L_2$

$$= P_3 \cdot \eta_{V3} \cdot \frac{\pi}{4} \cdot D_3^{\ 2} \cdot L_3 = ...$$

$$P_1 \cdot \eta_{V1} \cdot \frac{\pi}{4} \cdot D_1^{\ 2} \cdot L_1 = P_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} \cdot D_2^{\ 2} \cdot L_2$$

$$= P_3 \cdot \eta_{V3} \cdot D_3^{\ 2} \cdot L_3 = ...$$

If the volumetric efficiency is same for all cylinders, i.e $\eta_{V1} = \eta_{V2} = \eta_{V3} = ...$ and stroke for all cylinder is same i.e. $L_1 = L_2 = L_3 = ...$

Then, $D_1^{\ 2} P_1 = D_2^{\ 2} P_2 = D_3^{\ 2} P_3 = ...$

These generic relations may be used for getting the ratio of diameters of cylinders of multistage compression.

Energy balance: Energy balance may be applied on the different components constituting multistage compression.

For LP stage the steady flow energy equation can be written as below:

$$m \cdot h_1 + W_{LP} = m \cdot h_2 + Q_{LP}$$

$$Q_{LP} = W_{LP} - m(h_2 - h_1)$$
$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$

For intercooling (Fig. 5.5) between LP and HP stage steady flow energy equation shall be;

$$m \cdot h_2 = m \cdot h_{2'} + Q_{Int}$$
$$Q_{Int} = m(h_2 - h_{2'})$$
$$Q_{Int} = m \cdot C_p (T_2 - T_{2'})$$

For HP stage (Fig.5.5) the steady flow energy equation yields.

$$m \cdot h_{2'} + W_{HP} = m \cdot h_{3'} + Q_{HP}$$
$$Q_{HP} = W_{HP} + m(h_{2'} - h_{3'})$$
$$Q_{HP} = W_{HP} + m \cdot C_p (T_{2'} - T_{3'}) = W_{HP} - m \cdot C_p (T_{3'} - T_{2'})$$

In case of perfect intercooling and optimum pressure ratio, $T_{2'} = T_1$ and $T_2 = T_{3'}$

Hence for these conditions,

$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$
$$Q_{Int} = m \cdot C_p (T_2 - T_1)$$
$$Q_{HP} = W_{HP} - m \cdot C_p (T_2 - T_1)$$

Total heat rejected during compression shall be the sum of heat rejected during compression and heat extracted in intercooler for perfect intercooling.

Heat rejected during compression for polytropic process $= \left(\frac{\gamma - n}{\gamma - 1} \right) \times Work$

UNIT-5
Air Compressors

Review of equations

Work done in a single stage compressor

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] = \frac{n}{n-1} \; mR(T_2 - T_1)$$

Work done in a two stage compressor for perfect inter cooling

$$= \frac{2n}{n-1} P_1 V_1 \left[\left(\frac{P_3}{P_1} \right)^{\frac{n-1}{2n}} - 1 \right]$$

Work done in a two stage compressor

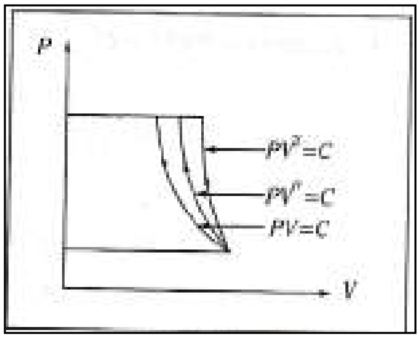
$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1} \left[1 + c - c \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$$

Volumetric Efficiency

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2 V_2 \left[\left(\frac{P_2}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

Problem 1

A single stage acting air compressor 30 cm bore and 40 cm stroke is running at a speed of 100 RPM. It takes in air at 1 bar and 20ºC and drive it when the compresses it to a pressure of 5 bar. Find the power required to drive it when compression is (i) isothermal (ii) PV1.2 = C and (iii) adiabatic. Also find the isothermal efficiencies for the cases (ii) and (iii) Neglect clearance.



$N_1 = 100RPM, d = 30cm$

$L = 40cm \qquad \mathrm{P}_1 = 1\,\mathrm{bar}$

$\mathrm{T}_1 = 20°C \qquad \mathrm{P}_2 = 5\,\mathrm{bar}$

$$V_s = \frac{\Pi}{4} d^2 L = \frac{\Pi}{4} 0.3^2 \chi \; 0.4$$

$$= 0.028\,\mathrm{m}^3 / cycle$$

$$V_1 = V_s = 0.028 \; \chi \; \frac{100}{60} = 0.047\,\mathrm{m}^3/s$$

$$m = \frac{\mathrm{P}_1 V_1}{RT_1} = \frac{1x10^2 \, x0.047}{0.287x293} = 0.055\,\mathrm{kg/s}$$

(i) Isothermal $\mathrm{PV} = \mathrm{C}$

$$\text{Power} = P_1V_1 Ln\frac{P_2}{P_1} = mRT_1 Ln\frac{P_2}{P_1}$$

$$= 0.055x0.287x293Ln\frac{5}{1} = 7.56kW$$

(i) $\mathrm{PV}^{1.2} = \mathrm{C}$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 293\left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = 383.14\,\mathrm{K}$$

$$\text{Power} = \frac{n}{n-1}\left(P_2V_2 - P_1V_1\right) = \frac{n}{n-1}\,x\,mR\left(T_2 - T_1\right)$$

$$= \frac{1.2}{1.2-1}\,x\,0.055\ \mathrm{x}\ 0.287\left(383.14 - 293\right) = 8.53\,\mathrm{kW}$$

$$Isothermaleffiency = \frac{\text{Isothermal Power}}{Actualpower}$$

$$= \frac{7.56}{8.53} = 0.8854$$

$$\eta_{Isothermal} = 88.54\,\%$$

(iii) Adiabatic $\mathrm{PV}^{1.4} = \mathrm{C}$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 293\left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 464\,\mathrm{K}$$

$$Isothermaleffiency = \frac{\text{Isothermal Power}}{Actualpower}$$

$$= \frac{7.56}{8.53} = 0.8854$$

$$\eta_{Isothermal} = 88.54\,\%$$

(iii) Adiabatic $\text{PV}^{1.4} = \text{C}$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 293\left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 464\,\text{K}$$

$$\text{Power} = \frac{\gamma}{\gamma-1}\,x\,mR(T_2 - T_1)$$

$$= \frac{1.4}{1.4-1}\,x\,0.055\text{ x }0.287(464-293) = 9.45\,\text{kW}$$

$$\eta_{Isothermal} = \frac{7.56}{9.45} = 0.80$$

$$= 80\,\%$$

Problem 2

A single acting stage acting air compressor with clearance running at 360 rpm has a bore of 10 cm. The compression and expansion are polytrophic with n = 1.25 for each. The clearance volume is 80cm3 . *If the suction and delivery pressures are 98.1 kPa and 706.32 kPa absolute, find the free air at 101 kPa and 15*ºCdelivered per minute. What is the work done per cycle? The temperature at the beginning of compression may be taken as 30ºC.Find also the power required to drive the compressor.

$T_a = 15°C,\qquad d = 10cm$

$L = 8.5cm \qquad \text{P}_1 = 98.1kPa$

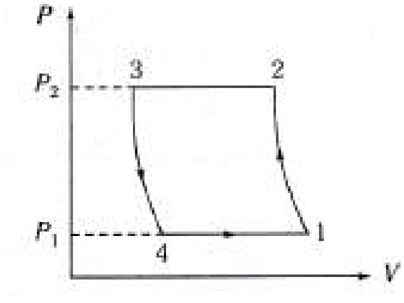
$\text{T}_1 = 30°C \qquad \text{P}_2 = 706.32kPa$

$\text{P}_a = 101kPa$

$$V_s = \frac{\Pi}{4}d^2L = \frac{\Pi}{4}0.1^2\,\chi\,0.85$$

$$= 6.675\text{ x }10^{-4}\text{ m}^3$$

$$V_c = 80cm = 0.8\,x\,10^{-4}\text{ m}^3$$



$$\textit{Clearance}\text{ ratio C} = \frac{\mathrm{V_c}}{\mathrm{V_s}} = \frac{0.8 \text{ x } 10^{-4}}{6.675 \text{ x } 10^{-4}} = 0.12$$

Volumetric efficiency

$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1}\left[1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$= \frac{98.1 \text{ x } 288}{101 \text{ x } 303}\left[1 + 0.12 - 0.12\left(\frac{706.32}{98.1}\right)^{\frac{1}{1.25}}\right] = 0.496$$

$$m = \frac{P_a T_a}{RT_a} = \frac{101 \text{ x } 0.1193}{0.287 \text{ x } 288}$$

$$= 0.1457 \text{ kg/min}$$

$$\eta_\upsilon = \frac{V_a}{V_s}$$

$$\therefore \text{Volume of free air } \mathrm{V_a} = 0.496 \text{ x } 6.675 \text{ x } 10^{-4}$$

$$= 0.0003314 \text{ m}^3 / \text{cycle}$$

$$= 0.0003314 \text{ x } 360 = 0.1193 \text{ m}^3 / \text{min}$$

$$\mathrm{m} = \frac{0.1457}{60} = 0.00242 kg / \text{sec}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 303\left(\frac{706.32}{98.1}\right)^{\frac{1.25-1}{1.25}} = 449.67 \text{ K}$$

$$\text{Power} = \left(\frac{\mathrm{n}}{n-1}\right) \text{x mR}(T_2 - T_1)$$

$$= \left(\frac{1.25}{1.25-1}\right) \text{x } 0.00242 \text{ x } 0.287(449.67 - 303)$$

$$= 0.5 \text{ kJ/s or kW}$$

$$Workdone / cycle = \frac{0.5}{360/60} = 0.0084 kJ / cycle$$

Problem 3

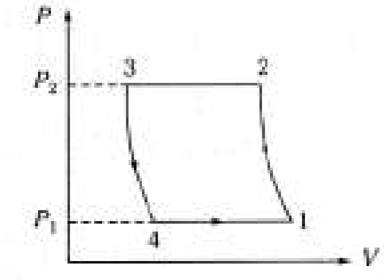
A single stage double acting air compressor is required to deal with 17 m3/min of air measured at 1 bar and 15°C. the pressure and temperature at the end of suction is 0.98 bar and 30°C. The delivery pressure is 6.3bar. The rpm of the compressor is 500.assuming a clearance volume of 5% of the stroke volume, laws of the compression and expansion as PV1.32 =C, calculate the necessary stroke of volume, temperature of the air delivered and power of the compressor.

$T_a = 15°C,\qquad V_a = 17 m^3/\min$

$P_a = 1bar \qquad P_1 = .98bar$

$T_1 = 30°C \qquad P_2 = 6.3bar$

$N = 500rpm$



$$C = \frac{5}{100} = 0.05$$

$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$\eta_\upsilon = \frac{98.1 \text{ x } 288}{1 \text{ x } 303}\left[1 + 0.05 - 0.05\left(\frac{6.3}{0.98}\right)^{\frac{1}{1.32}}\right] = 0.8452$$

$$\eta_\upsilon = \frac{V_a}{V_s}$$

$$V_s = StrokeVolume = \frac{17}{0.8452} = 20.11 m^3/\min$$

$$V_s = \frac{20.11}{500} = 0.0402 m^3/\min$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 303\left(\frac{6.3}{0.98}\right)^{\frac{1.32-1}{1.32}}$$

$$Temperature \text{ of air delivered} = 475.71\,\text{K}$$

$$m = \frac{P_a T_a}{R T_a} = \frac{1 \text{ x } 100 \text{ x } 17}{0.287 \text{ x } 288} = 20.56\text{kg/min}$$

$$= \frac{20.56}{60} = 0.3427 \text{ kg/sec}$$

$$\text{Power} = \left(\frac{n}{n-1}\right) \text{ x mR}\left(T_2 - T_1\right)$$

$$= \left(\frac{1.32}{1.32-1}\right) \text{ x } 0.3427 \text{ x } 0.287\left(495.71 - 303\right)$$

$$= 70 \text{ kJ/s or kW}$$

Problem 4

A single stage double acting air compressor delivers 15 m3 of air per min of air measured at 1.013 bar and 27°C. delivers at 7bar.The condition at the end of the suction stroke are pressure 0.98 bar and temperature 4°C. The a clearance volume is 4% of the swept volume, and stroke to bore ration is 1.3:1 and compressor runs at 300rpm. calculate the Volumetric efficiency of the compressor. Assume the index of compression and expansion to be 1.3.

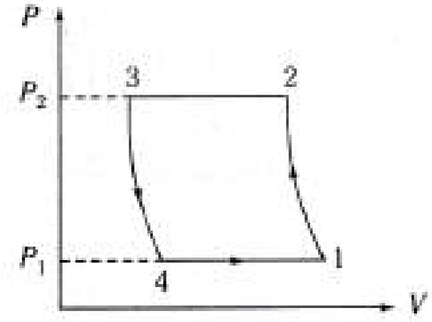
$T_a = 27°C, \qquad V_a = 15 m^3/\text{min}$

$P_a = 1.013 bar \qquad P_1 = .98 bar$

$T_1 = 40°C \qquad P_2 = 7 bar$

$N = 300 rpm, Clearanceratio = 0.04$

$L/d = 1.3$



$$\eta_v = \frac{P_1 T_a}{P_a T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$\eta_v = \frac{98.1 \text{ x } 300}{1.013 \text{ x } 313}\left[1 + 0.04 - 0.04\left(\frac{7}{0.98}\right)^{\frac{1}{1.3}}\right] = 0.796$$

$$= 79.6\%$$

$$\eta_v = \frac{V_a}{V_s}$$

$$V_s = Swept\,Volume = \frac{15}{0.796} = 18.84\, m^3/\text{min}$$

But for double acting compressor

$$V_s = \frac{\Pi}{4} d^2 \text{ x L } 2NL = 1.3d$$

$$18.84 = \frac{\Pi}{4} d^2 \text{ x } 1.3d \text{ x } 2 \text{ x } 300$$

$$d = \sqrt[3]{\frac{18.8 \text{ x } 44}{\Pi \text{ x } 1.3 \text{ x } 2 \text{ x } 300}} = 0.313 m = 31.3 cm$$

$$L = 1.3 x 31.3 = 40.72 \text{cm}$$

$$m = \frac{P_a T_a}{RT_a} = \frac{1.013 \text{ x } 100 \text{ x } 15}{0.287 \text{ x } 300} = 17.6 \text{kg/min}$$

$$= \frac{17.6}{60} = 0.294 \text{ kg/s}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 313 \left(\frac{7}{0.98} \right)^{\frac{1.3-1}{1.3}} = 492.7 \text{ K}$$

$$\text{Power} = \left(\frac{\text{n}}{n-1} \right) \text{x mR} \left(T_2 - T_1 \right)$$

$$\text{Power} = \left(\frac{1.3}{1.3-1} \right) \text{x } 0.294 \text{ x } 0.287 \left(492.71 - 313 \right)$$

$$= 65.7 \text{ kJ/s or kW}$$

$$\text{Isothermal Power} = P_1 V_1 Ln \frac{P_2}{P_1} = mRT_1 Ln \frac{P_2}{P_1}$$

$$= 0.294 x 0.287 x 313 Ln \frac{7}{0.98} = 51.92 kW$$

$$Isothermal \text{ efficiency} = \frac{51.92}{65.7} x 100 = 79\%$$

Problem 5

A two-stage compressor compresses 1kg/min of air from 1bar to 42.18 bar. Initial temperature is 15ºC. At the intermediate pressure the intercooling is perfect. The compression takes place according to PV1.35 =C. Neglecting the effect of clearance, determine the minimum power required to run the compressor. Also find the mass of cooling water required in the intercooler , if the temperature rise of water is limited to 5°C.

$$M = 1 kg / \min = \frac{1}{60} = 0.0166 kg/s$$

$$P_1 = 1 bar \quad \text{T}_1 = 15°C \quad \text{P}_3 = 42.18 bar$$

$$For \text{ perfect intercooling}$$

$$\text{P}_2 = \sqrt{\text{P}_1 \text{P}_3} = \sqrt{1 \text{x} 42.18} = 6.48 bar$$

$$\text{T}_2 = \text{T}_1 \left(\frac{\text{P}_2}{\text{P}_1} \right)^{\frac{n-1}{n}} = 288 \left(\frac{6.49}{1} \right)^{\frac{1.35-1}{1.35}} = 467.7 K$$

$$\text{Maximum Power} = \left(\frac{\text{n}}{n-1}\right) \text{x mR}\left(T_2 - T_1\right)$$

$$= \left(\frac{2\text{x}1.35}{1.35-1}\right) \text{x } 0.0166 \text{ x } 0.287\left(467.7 - 288\right)$$

$$= 6.6 \text{ kJ/s or kW}$$

$$\text{Heat rejected in the intercooler} \quad \text{Q} = \text{mC}_\text{p}\left(T_2 - T_1\right)$$

$$= 0.0166x1.005\left(467.7 - 288\right)$$

$$= 2.99\text{kJ/Sec}$$

$$\textit{But } \text{Q is also} = \text{m}_\omega x C_{p\omega} x \text{ temperature rise}$$

$$299 = \text{m}_\omega x4.187 \text{ x } 5$$

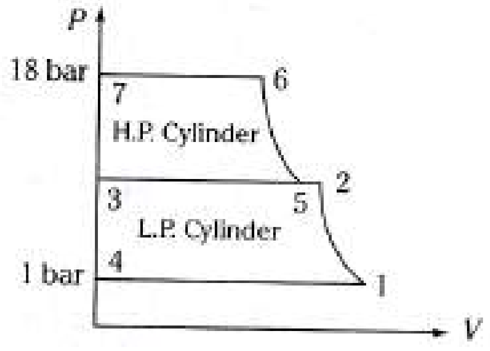
$$\text{m}_\omega = 0.1432kg/\text{sec}$$

Problem 6

A two stage reciprocating compressor delivers 150 m3/hr of free air measured at 1.03 bar and15ºC. The final pressure 18bar. The pressure and temperature of the air in LP cylinder before compression is 1 bar and 30°C. the diameter of the LP cylinder is twice that of HP cylinder and air enters the HP cylinder at 40ºC. If compression follows the law PV1.22=C, determine

(i) Intermediate pressure and power required if the intercooler is imperfect.

(ii) Ration of cylinder diameter and minimum power required for perfect intercooling.



$T_a = 15°C, \quad \text{V}_a = 150m^3/\text{min}$

$\text{P}_\text{a} = 1.03bar \quad \text{P}_1 = 1bar$

$\text{T}_1 = 30°C \quad \text{P}_3 = 18bar$

$\text{T}_2 = 40°C$

Neglet the effect of clearance

(i) Imperfect intercooling

$\text{d}_\text{LP} = 2d_{HP}$

$P_1V_1^{1.22} = P_2V_2^{1.22}$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{1.22} = \left(\frac{\Pi/4xd_{LP}^2xL}{\Pi/4xd_{HP}^2xL}\right)^{1.22}$$

$$\frac{P_2}{P_1} = \left(\frac{d_{LP}}{d_{HP}}\right)^{2x1.22}$$

$P_2 = 1x(2)^{2.44}$

$\textit{Intercooling pressure } \text{P}_2 = 5.426bar$

$$\frac{P_1V_1}{T_1}=\frac{P_aV_a}{T_a}$$

$$V_1=\frac{P_aV_aT_1}{T_aP_1}=\frac{1.03x10^2\,x0.02916x303}{1x10^2\,x288}$$

$$=0.045\mathrm{m}^3/\mathrm{sec}$$

$$W_{LP}=\frac{n}{n-1}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$W_{LP}=\frac{1.22}{1.22-1}1x10^2\,x0.045\left[\left(\frac{5.426}{1}\right)^{\frac{1.22-1}{1.22}}-1\right]$$

$$=8.89\,\mathrm{kW}$$

Air enters HP cylinder at $\mathrm{T}_2=40°C$

$$\frac{P_2V_2}{T_2}=\frac{P_1V_1}{T_1}$$

$$V_2=\frac{P_1V_1T_2}{T_1P_2}=\frac{1x10^2\,x0.045x313}{5.426x10^2\,x303}$$

$$=0.008567\mathrm{m}^3/\mathrm{sec}$$

$$W_{HP}=\frac{n}{n-1}P_2V_2\left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}-1\right]$$

$$W_{HP}=\frac{1.22}{1.22-1}5.426x10^2\,x0.008567\left[\left(\frac{18}{5.426}\right)^{\frac{1.22-1}{1.22}}-1\right]$$

$$=6.12\,\mathrm{kW}$$

$$\text{Total Power}=8.89+6.12$$

$$=15.01\,\mathrm{kW}$$

Problem 7

A multi stage compressor compressing air is to be designed to elevate the pressure from 1 bar to 120bar such that the stage pressure ratio should not exceed 4. Determine

(i) The number of stages
(ii) Exact stage pressure ratio
(iii) Intermediate pressure

Solution:

$$P_1 = 1 bar \quad P_{N+1} = 120 bar$$

$$Stage \text{ pressure ratio} = \frac{P_2}{P_1} = \frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \frac{P_{N+1}}{P_N} = 4$$

Assuming the intercooling to be perfect we have

$$\frac{P_{N+1}}{P_N} = \left(\frac{P_{N+1}}{P_1}\right)^{\frac{1}{N}}$$

$$4 = \left(\frac{120}{1}\right)^{\frac{1}{N}}$$

$$N = \frac{Ln120}{Ln4} = 3.453$$

$\therefore$ Number of stages = 4

$$\text{Exact stage pressure ratio} = \frac{P_{N+1}}{P_N} = (120)^{\frac{1}{4}} = 3.31$$

$$\frac{P_5}{P_4} = 3.31, \ P_4 = \frac{120}{3.310} = 36.25\, bar$$

$$\frac{P_4}{P_3} = 3.31, \ P_3 = \frac{36.25}{3.31} = 10.95\, bar$$

$$\frac{P_3}{P_2} = 3.31, \ P_2 = \frac{10.95}{3.31} = 3.308\, bar$$

$\therefore$ Intermediat pressure are 36.25 bar, 10.95 bar and 3.308 bar

Problem 8

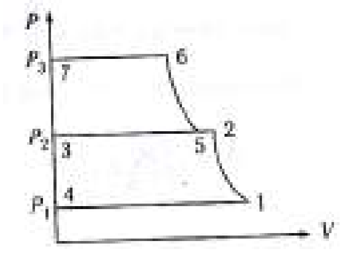
A two-stage compressor delivers air at pressure of 19 bar. The free air conditions are 1.03 bar and 25ºC. The pressure of the air before compression is 0.98bar. The intermediate pressure is 4.5bar. The temperature of the air entering each cylinder is 35°C. The law of compression and expansion being PV1.25 =C. The clearance volume is 5% of the swept volume. Determine the volumetric efficiency and the work done per kg of air.

$T_a = 25°C, \qquad P_2 = 4.5 bar$

$P_a = 1 bar \qquad P_1 = 0.98 bar$

$T_1 = 35°C \qquad P_3 = 19 bar$

$Clearance\ ratio\ C = 0.05$



$$\eta_\upsilon = \frac{P_1 T_a}{P_a T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$\eta_\upsilon = \frac{98.1\text{ x }298}{1\text{ x }308}\left[1 + 0.05 - 0.05\left(\frac{4.5}{0.98}\right)^{\frac{1}{1.25}}\right] = 0.835$$

$$= 83.5\%$$

Even through the temperature of air entering each cylinder is same, work is not saem in both cylinder since $\mathrm{P}_2 \neq \sqrt{P_1 P_3}$

$$\mathrm{T}_2 = \mathrm{T}_1\left(\frac{\mathrm{P}_2}{\mathrm{P}_1}\right)^{\frac{n-1}{n}} = 308\left(\frac{4.5}{0.98}\right)^{\frac{1.25-1}{1.25}} = 416.1K$$

$$W_{LP} = \frac{n}{n-1}R(T_2 - T_1) = \frac{1.25}{1.25-1}x0.287(416.1 - 308)$$

$$= 155.12\text{ kJ/kg}$$

Temperature of air entering HP cylinder $\mathrm{T}_5 = 35°\mathrm{C}$

$$\mathrm{T}_6 = \mathrm{T}_5\left(\frac{\mathrm{P}_6}{\mathrm{P}_5}\right)^{\frac{n-1}{n}} = 308\left(\frac{19}{4.5}\right)^{\frac{1.25-1}{1.25}} = 410.82K$$

$$W_{HP} = \frac{n}{n-1}R(T_6 - T_5) = \frac{1.25}{1.25-1}0.287(410.82 - 308)$$

$$= 147.55\text{kJ/kg}$$

Total work $= 155.12 + 147.55$

$= 302.75$kJ/kg of air

Problem 9

A two-stage double acting air compressor takes in air at 1 bar and 25°C. It runs at 200 rpm. The diameter of LP cylinder is 35cm. The stroke of both LP and HP cylinders are 40cm. The clearance volume of both the cylinders is 4%. The index of compression is 1.3. The LP cylinder discharges air at a pressure of 4 bar. The air passes through the intercooler so that it enters the HP cylinder at 27°C and 3.6 bar, finally it is discharged from the compressor at 14.4 bar.

Calculate

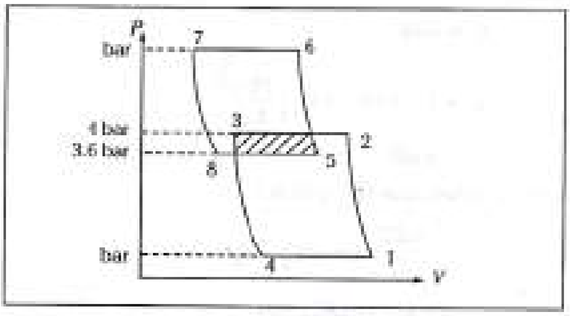
(i) Diameter of HP cylinder
(ii) The heat rejected in the intercooler
(iii) The power required to drive the HP cylinder

$P_1 = 1$ bar $\quad T_1 = 25°C \quad N = 200$ For a double acting compressor, swept volume of LP cylinder

$$= \frac{\pi}{4} D_{LP}^2 \times L_{LP} \times \frac{2N}{60}$$

$$= \frac{\pi}{4} \times 0.35^2 \times 0.4 \times \frac{2 \times 220}{60}$$

$$V_s = 0.2822 \ m^3/\text{sec}$$



Volumatric effiency referred to the suction condition 1

$$\eta_\upsilon = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

Volume of air referred to condition 1

$$V_1 = 0.9238x0.2822 = 0.26071 m^3/S$$

$$\textit{Mass of air} \quad m = \frac{P_1V_1}{RT_1} = \frac{1x10^2x0.26071}{0.287x298}$$

$$= 0.30483 \text{ kg/s}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 298\left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}} = 410.34K$$

Heat rejected in the intercooler

$$= mC_p(T_2 - T_1) = 0.30483x1.005(410.34 - 298)$$

$$= 34.14 \text{kJ/s}$$

Volume of air draw to HP cylinder

$$V_5 = \frac{mRT_5}{P_5} = \frac{0.30483x0.287x298}{3.6x10^2} = 0.0724 m^3/s$$

Pressure ratio in LP cylinder = 4
Pressure ratio in HP cylinder =14.4/3.6=4
Since the pressure ratio and clearance percentage in both HP and LP cylinders are the same, the volumetric efficiency of the cylinders referred to the condition at the start of compression is same.

$$\therefore \text{Swept volume of HP cylinder} = \frac{V_5}{\eta_\upsilon} = \frac{0.0724}{0.9238}$$

$$= 0.07837 \text{ m}^3/s$$

$$\frac{\Pi}{4} D^2{}_{HP} x L_{HP} x \frac{2x200}{60} = 0.7837$$

$$\frac{\Pi}{4} D^2{}_{HP} x 0.4 x \frac{2x200}{60} = 0.1934 m$$

$$= 19.34 \text{ cm}$$

Since pressure ratio is same $T_6 = T_2$

Power required for HP cylinder

$$= \left(\frac{n}{n-1}\right) \text{x mR}(T_6 - T_5)$$

$$= \left(\frac{n}{n-1}\right) \text{x mR}(T_2 - T_1)$$

$$= \left(\frac{1.3}{1.3-1}\right) \text{x } 0.30483x0.287(410.34 - 298)$$

$$= 42.58 \text{ kW}$$

Problem 10

A two stage air compressor compresses air from 17^0C and 1 bar to 63 bar. The air is cooled in the intercooler to 30^0C and intermediate pressure is steady at 7.7 bar. The low pressure cylinder is 10 cm diameter and the stroke for both cylinders is 11.25 cm. Assuming a compression law of $PV^{1.35}$ = constant, and that the volume of air at atmospheric conditions drawn in per stroke is equal to the low pressure cylinder swept volume, find the power of the compressor while running at 250 rpm. Find also the diameter of HP cylinder.

Solution:

d_{LP} = 10cm L = 11.25cm
P_1 = 1 bar P_2 = 7.7 bar $T_1 = 17^0C$
P_3= 63 bar

Volume of LP cylinder

$V_1 = \pi/4 \text{ X } 0.1^2 \text{ X } 0.1125 = 0.00088 \text{ m}^3$

$m = P_1V_1/R\, T_1 = (1 \text{ X } 100 \text{ X } 0.00088)/(0.287 \text{ X } 290) = 0.00106 \text{ kg}$

$T_2 = 30^0C$

Volume of air entering the HP cylinder

$V_2 = mR\, T_2/ P_2 = (0.00106 \text{ X } 0.283 \text{ X } 303)/ (7.7 \text{ X } 10^2) = 0.0001198 \text{ m}^3$

$V_2 = \pi/4 \text{ X } d_2^{\,2} \text{ X } L$
$0.0001198 = \pi/4 \text{ X } d_2^{\,2} \text{ X } 0.1125$
d_2 = diameter of HP cylinder
= 0.0368 m

Diameter of HP cylinder = 3.68 cm

Work required/cycle = $(n/n\text{-}1)[\, P_1V_1\{(P_2 / P_1)^{n-1/n} - 1\}+ P_2V_2(P_3 / P_2)^{n-1/n} - 1\,]$

$W = (1.35/1.35\text{-}1)\, [\, 1 \text{ X } 10^2 \text{ X } 0.00088 \{(7.7/1)^{1.35-1/1.35} - 1\} + 7.7 \text{ X } 10^2 \text{ X } 0.0001198 \{(63/7.7)^{1.35-1/1.35} - 1\}]$

= 0.49456 kJ/cycle

Power = 0.49456 X Number of cycles/sec
= 0.49456 X 250/60
Power = 2.06 kW

Problem 11

A three stage air compressor draws 8 m^3/min of air at 1 bar and 18^0C and delivers the same at 55 bar and 20^0C. The index of compression is 1.32. The air while passing through the intercoolers and aftercoolers suffers a pressure loss of 4% and is cooled to the intial temperature. Determine the shaft power required to drive the compressor if mechanical efficiency is 85%.

Solution:

$V_1 = 8 \text{ m}^3/\text{min} = 8/60 = 0.133 \text{ m}^3/\text{sec}$
P_1 = 1 bar $T_1 = 18^0C$ P_4 = 55 bar

The pressure drop of 4% in the intercooler is accounted for by the factor C = 0.96

For 3 stage compressor,

Power = $3(n/n-1) \times P_1V_1 [\{P_4/C^3P_1\}^{n-1/3n} - 1]$

$= 3\ (1.32/1.32-1) \times 1 \times 10^2 \times 0.133\ [\{55/0.96^3 \times 1\}^{1.32-1/1.32} - 1]$

= 283.kW

Actual shaft power required = Power/η_{mech} = 283.32/0.85 = **333.32kW**

STEAM NOZZLES

8.1 Introduction

In the impulse steam turbine, the overall transformation of heat into mechanical work is accomplished in two distinct steps. The available energy of steam is first changed into kinetic energy, and this kinetic energy is then transformed into mechanical work. The first of these steps, viz., the transformation of available energy into kinetic energy is dealt with in this chapter.

A nozzle is a passage of varying cross-sectional area in which the potential energy of the steam is converted into kinetic energy. The increase of velocity of the steam jet at the exit of the nozzle is obtained due to decrease in enthalpy (total heat content) of the steam. The nozzle is so shaped that it will perform this conversion of energy with minimum loss.

8.2 General Forms of Nozzle Passages

A nozzle is an element whose primary function is to convert enthalpy (total heat) energy into kinetic energy. When the steam flows through a suitably shaped nozzle from zone of high pressure to one at low pressure, its velocity and specific volume both will increase.

The equation of the continuity of mass may be written thus :

...(8.1)

where m m mass flow in kg/sec.,

V = velocity of steam in m/sec.,

A = area of cross-section in m , and

v = specific volume of steam in m3/kg.

In order to allow the expansion to take place properly, the area at any section of the nozzle must be such that it will accomodate the steam whatever volume and velocity may prevail at that point.

As the mass flow (m) is same at all sections of the nozzle, area of cross-section (A) varies as —. The manner in which both V and v vary depends upon the properties of the substance flowing. Hence, the contour of the passage of nozzle depends upon the nature of the substance flowing.

For example, consider a *liquid*- a substance whose specific volume v remains almost constant with change of pressure. The value of will go on increasing with change of pressure. Thus, from eqn. (8.1), the area of cross-section should decrease with the decrease of pressure. Fig. 8-1 (a) illustrates the proper contour of longitudinal section of a nozzle suitable for liquid. This also can represent convergent nozzle for a fluid whose peculiarity is that while both velocity and specific volume increase, the rate of specific volume increase is less than that of the velocity, thus resulting in increasing value of $\frac{V}{v}$.

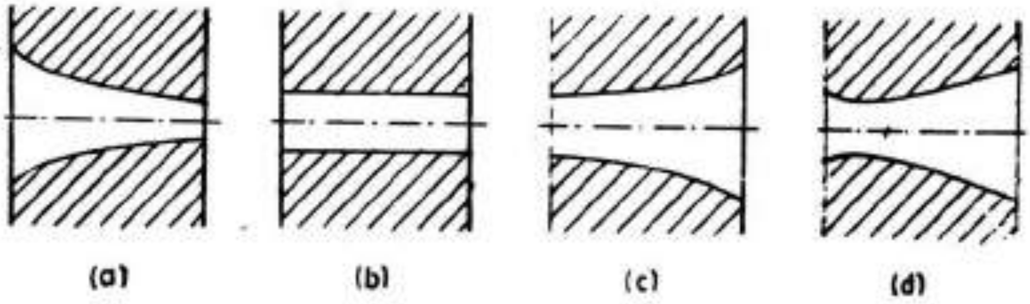

(a) (b) (c) (d)

Fig. 8-1. General forms of Nozzles.

Fig. 8-1(b) represents the correct contour for some hypothetical substance for which both velocity and specific volume increase at the same rate, so that their ratio $\frac{V}{v}$ is a constant at all points. The area of cross-section should therefore, be constant at all points, and the nozzle becomes a plain tube.

Fig. 8-1(c) represents a divergent nozzle for a fluid whose peculiarity is that $\frac{V}{v}$ decreases with the drop of pressure, i.e., specific volume increases at a faster rate than velocity with the drop of pressure. The area of cross-section should increase as the pressure decreases.

Table 8-1

Properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0.15 bar through a nozzle, assuming frictionless adiabatic flow.

Pressure p bar	Dryness fraction x	Enthalpy drop $H_1 - H_2$ kJ	Velocity V m/sec.	Specific Volume v_s m^3/kg	Discharge per unit area kg/m^2	Area A m^2	Diameter D metre
14	1·000	-	-	-	-	-	-
12	0·988	38·6	278	0·1633	1,723	0·00058	0·0272
10	0.974	84·1	410	0·1944	2,165	0·00046	0·0242
7	0·950	164·7	574	0·2729	*2,214	0·00045	x0·0239
3·5	0·908	309	786	0·5243	1,651	0·00061	0·0279
1·5	0·872	441·2	939	1·1593	929	0·0011	0·0374
0·70	0·840	555·6	1,054	2·365	531	0·00188	0·049
0·15	0·790	736·7	1,214	10·022	153	0·0065	0·091

* Maximum discharge per unit area x Smallest diameter

Fig. 8-1(d) shows the general shape of convergent-divergent nozzle suitable for gases and vapours. It can be shown that in practice, while velocity and specific volume both increase from the start, velocity first increases faster than the specific volume, but after a certain critical point, specific volume increases more rapidly than velocity. Hence the value of $\frac{V}{v}$ first increases to maximum and then decreases, necessitating a nozzle of *convergent-divergent* form. The above statement may be verified by referring to table 8-1, which shows the properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0·15 bar through a nozzle, assuming frictionless adiabatic flow.

8.3 Steam Nozzles

The mass flow per second for wet steam, at a given pressure during expansion is given by

$$m = \frac{AV}{v} = \frac{AV}{xv_s} \text{ kg/sec.} \qquad \text{...(8.2)}$$

where A = Area of cross-section in m^2,

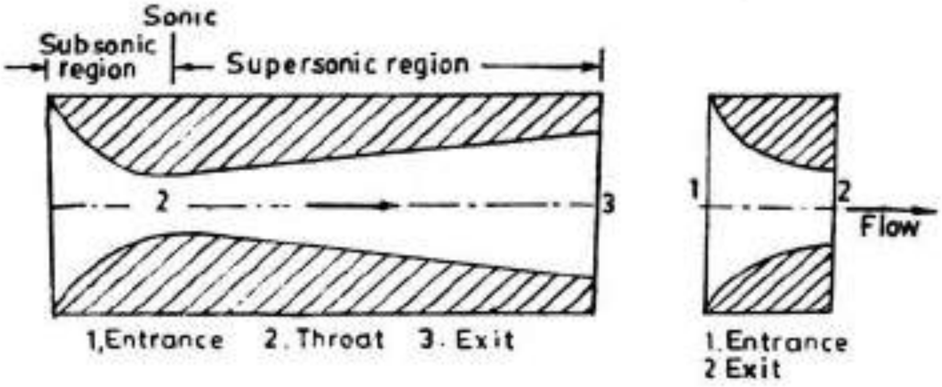
V = Velocity of steam in m/sec,

v_s = Specific volume of dry saturated steam, m^3/kg,

x = Dryness fraction of steam, and

$v = x\ v_s$ = Specific volume of wet steam, m^3/kg.

As the mass of steam per second (m) passing through any section of the nozzle must be constant, the area of cross-section (A) of nozzle will vary according to the variation of $\frac{V}{xv_s}$ i.e., product of A and $\frac{V}{xv_s}$ is constant. If the factor $\frac{V}{xv_s}$ increases with the drop in pressure, the cross-sectional area should decrease and hence a *convergent* shaped nozzle. The decrease of the factor $\frac{V}{xv_s}$ with pressure drop will require increasing cross-sectional area to maintain mass flow constant and hence the *divergent* shaped nozzle.



(a) Converging–diverging nozzle (b) Converging nozzle

Fig. 8-2. Longitudinal sectional view of steam nozzles.

In practice at first the nozzle cross-section tapers to a smaller section in order to allow for increasing value of $\frac{V}{xV_s}$; after this smallest diameter is reached, it will diverge to a larger cross-section. The smallest section of the nozzle is known as the *throat.*

A nozzle which first converges to throat and then diverges, as in fig. 8-2(a), is termed as *converging-diverging nozzle.* It is used for higher pressure ratio $\left(\frac{p_2}{p_1}\right)$.

Some form of nozzles finish at the throat and no diverging portion is fitted; this type shown in fig. 8-2(b), is known as *converging nozzle.* In this the greatest area is at the entrance and minimum area is at the exit which is also the throat of the nozzle. This nozzle is used when the pressure ratio, $\frac{P_2}{p_1}$ is less than 0.58 (critical).

8.4 Flow Through Steam Nozzles

From the point of view of thermodynamics, the steam flow through nozzles may be spoken as adiabatic expansion. During the flow of steam through the nozzle, heat is neither supplied nor rejected. Moreover, as the steam expands from high pressure to low pressure, the heat energy is converted into kinetic energy, i.e., work is done in expanding to increase the kinetic energy. Thus the expansion of steam through a nozzle is an adiabatic, and the flow of steam through nozzle is regarded as an adiabatic flow.

It should be noted that the expansion of steam through a nozzle is not a free expansion, and the steam is not throttled, because it has a large velocity at the end of the expansion. Work is done by the expanding steam in producing this kinetic energy.

In practice, some kinetic energy is lost in overcoming the friction between the steam and the side of the nozzle and also internal friction, which will tend to regenerate heat. The heat thus formed tends to dry the steam. About 10% to 15% of the enthalpy drop from inlet to exit is lost in friction. The effect of this friction, in resisting the flow and in drying the steam, must be taken into account in the design of steam nozzles, as it makes an appreciable difference in the results.

Another complication in the design of steam through a nozzle is due to a phenomenon known as *supersaturation;* this is due to a time lag in the condensation of the steam during the expansion. The expansion takes place very rapidly and if the steam is initially dry or superheated, it should become wet as the pressure falls, because the expansion is adiabatic. During expansion the steam does not have time to condense, but remains in an unnatural dry or superheated state, then at a certain instant, it suddenly condenses to its natural state. See illustrative problem no. 14.

Thus, the flow of steam through a nozzle may be regarded as either an ideal adiabatic (isentropic) flow, or adiabatic flow modified by friction and supersaturation.

If friction is negligible, three steps are essential in the process of expansion from pressure p_1 to p_2 :

(i) Driving of steam upto the nozzle inlet from the boiler. The 'flow-work' done on the steam is p_1v_1 and results in similar volume of steam being forced through the exit to make room for fresh charge (steam).

(ii) Expansion of steam through the nozzle while pressure changes from p_1 to p_2, the work done being $\frac{1}{n-1}(p_1v_1 - p_2v_2)$

where n is the index of the isentropic expansion,

v_1 = volume occupied by 1 kg of steam at entrance to nozzle, and

v_2 = volume occupied by 1 kg of steam as it leaves the nozzle.

Alternatively, this work done is equal to the change of internal energy, $\mu_1 - \mu_2$ as during isentropic expansion work is done at the cost of internal energy.

(iii) Displacement of the steam from the low pressure zone by an equal volume discharged from the nozzle. This work amounts to p_2v_2 which is equal to the final flow work spent in forcing the steam out to make room for fresh charge (steam).

Thus, the new work done in increasing kinetic energy of the steam,

$$W = p_1v_1 + \left[\frac{1}{n-1}(p_1v_1 - p_2v_2)\right] - p_2v_2$$

$$W = \frac{n}{n-1}(p_1v_1 - p_2v_2) \qquad \text{... (8.3)}$$

This is same as the work done during Rankine cycle.

Alternatively, $W = p_1v_1 + (\mu_1 - \mu_2) - p_2v_2$

$$= (p_1v_1 + \mu_1) - (\mu_2 + p_2v_2) = H_1 - H_2 \qquad \text{...(8.4)}$$

where, H_1 and H_2 are the values of initial and final enthalpies allowing for the states of superheating or wetness as the case may be. This is exactly equivalent to the enthalpy drop equivalent to the work done during the Rankine cycle. The value of $H_1 - H_2$ may be found very rapidly from the Mollier chart ($H - \Phi$ chart) or more slowly but with greater accuracy from the steam tables.

In the design of steam nozzles the calculations to be made are :

(i) the actual velocity attained by the steam at the exit,

(ii) the minimum cross-sectional area (throat area) required for a given mass flow per second,

(iii) the exit area, if the nozzle is converging-diverging, and

(iv) the general shape of the nozzle – axial length.

8.4.1 Velocity of steam leaving nozzle : The gain of kinetic energy is equal to the enthalpy drop of the steam. The initial velocity of the steam entering the nozzle (or velocity of approach) may be neglected as being relatively very small compared with exit velocity.

For isentropic (frictionless adiabatic) flow and considering one kilogram of steam

$$\frac{V^2}{2 \times 1{,}000} = H_1 - H_2 = H$$

where H is enthalpy drop in kJ/kg and V = velocity of steam leaving the nozzle in m/sec.

$$\therefore\ V = \sqrt{2 \times 1{,}000H} = 44{\cdot}72\sqrt{H}\ \text{m/sec.} \qquad \text{... (8.5)}$$

Let the available enthalpy drop after deducting frictional loss be kH,

i.e. $(1 - k)\,H$ is the friction loss,

Then, $V = 44{\cdot}72\sqrt{kH}$ m/sec. ... (8.6)

If the frictional loss in the nozzle is 15 per cent of the enthalpy drop, then $k = 0.85$.

8.4.2 Mass of steam discharged : The mass flow of steam in kg per second through a cross-sectional area A and at a pressure p_2 can be written as

$m = \frac{AV_2}{v_2}$ where v_2 = specific volume of steam at pressure p_2.

But $v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = v_1 \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$ ...(8.7)

where, v_1 = specific volume of steam at pressure p_1.

Using the value of velocity V from eqns. (8.3.) and (8.5),

$$m = \frac{A}{v_2}\sqrt{\left[2{,}000\frac{n}{n-1}(p_1v_1 - p_2v_2)\right]} = \frac{A}{v_2}\sqrt{2{,}000\frac{n}{n-1}p_1v_1\left[1 - \frac{p_2v_2}{p_1v_1}\right]}$$

Putting the value of v_2 from eqn. (8.7), we get,

$$m = \frac{A}{v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}}\sqrt{2{,}000\frac{n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$m = A\sqrt{2{,}000\frac{n}{n-1} \times \frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]} \quad \text{... (8.8)}$$

8.4.3 Critical pressure ratio : Using eqn. (8.8), the rate of mass flow per unit area is given by

$$\frac{m}{A} = \sqrt{2{,}000\frac{n}{n-1} \times \frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$$

The mass flow per unit area has the maximum value at the throat which has minimum area, the value of pressure ratio $\left(\frac{p_2}{p_1}\right)$ at the throat can be evaluated from the above expression corresponding to the maximum value of $\frac{m}{A}$.

All the items of this equation are constant with the exception of the ratio $\left(\frac{p_2}{p_1}\right)$.

Hence, $\frac{m}{A}$ is maximum when $\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]$ is the maximum.

Differentiating the above expression with respect to $\left(\frac{p_2}{p_1}\right)$ and equating to zero for a maximum discharge per unit area

$$\frac{d}{d\left(\frac{p_2}{p_1}\right)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right] = 0$$

$$\therefore \frac{2}{n}\left[\frac{p_2}{p_1}\right]^{\frac{2}{n}-1} - \frac{n+1}{n}\left[\frac{p_2}{p_1}\right]^{\frac{n+1}{n}-1} = 0$$

Hence, $\left[\frac{p_2}{p_1}\right]^{\frac{2-n}{n}} = \frac{n+1}{n}\left[\frac{p_2}{p_1}\right]^{\frac{1}{n}}$ or $\left[\frac{p_2}{p_1}\right]^{2-n} = \left[\frac{n+1}{2}\right]^{n}\left(\frac{p_2}{p_1}\right)$

from which $\left[\frac{p_2}{p_1}\right]^{1-n} = \left[\frac{n+1}{2}\right]^{n}$ or $\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$ ... (8.9)

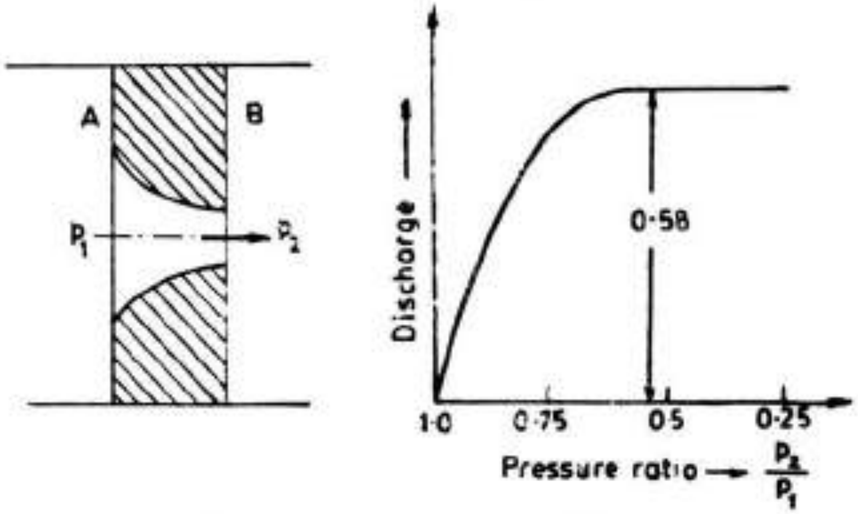
$\frac{p_2}{p_1}$ is known as *critical pressure ratio* and depends upon the value of index *n*.

The following approximate values of index *n* and corresponding values of critical pressure ratios may be noted :

Initial condition of steam	Value of index *n* for isentropic expansion	Nozzle critical pressure ratio $\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$
Superheated or supersaturated	1·300	0·546
Dry saturated	1·135	0·578
Wet	1·113	0·582

Dr. Zeuner has suggested a well known equation for value of *n* in the adiabatic expansion of steam viz. $n = 1{\cdot}035 + 0{\cdot}1x_1$, where x_1 is the initial dryness fraction of steam.

The eqn. (8.9) gives the ratio between the throat pressure (p_2) and the inlet pressure (p_1) for a maximum discharge per unit area through the nozzle. The mass flow being constant for all sections of nozzle, maximum discharge per unit area occurs at the section having minimum area, i.e., at the throat. The area of throat of all steam nozzle should be designed on this ratio. This pressure ratio at the throat is known as *critical pressure ratio. The pressure at which the area is minimum and discharge per unit area is maximum is termed as the critical pressure.*



(a) (b)

Fig. 8-3.

The implication of the existence of a critical pressure in nozzle flow may be expressed in another way. Suppose we have two vessels A and B. A containing steam at a high and steady pressure p_1. Suppose that the pressure in B may be varied at will. A and B are connected by a diaphragm containing a convergent nozzle, as shown in fig. 8-3(a).

Assume at first that p_2 is equal to p_1, then there is no flow of steam through the nozzle. Now let p_2 be gradually reduced. The discharge *m* through the nozzle will increase as shown by the curve of fig. 8-3(b). As the pressure p_2 approaches the critical value, the discharge rate gradually approaches its maximum value, and when p_2 is reduced below the critical value, the discharge rate does not increase but remains at the same value as that at the critical pressure. The extraordinary result that p_2 can be reduced well below the critical pressure without influencing the mass flow was first discovered by R.D. Napier.

Another explanation can be visualised as follows : the critical pressure will give velocity of steam at the throat equal to the velocity of the sound *(sonic velocity)*. The flow of steam in the convergent portion of the nozzle is *sub-sonic*. Thus, to increase the velocity of steam above sonic velocity *(super sonic)* by expanding steam below critical pressure, divergent portion is necessary [fig· 8-2(a)]·